Developing the Essential Strategies for Computation

PROFESSIONAL LEARNING PAPER

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As President of ORIGO Education, James strives to lift the profile of mathematics through dynamic professional development and the development of quality, research-based classroom materials. James frequently presents workshops and speaks at conferences throughout North America, Australia, and New Zealand and has authored more than 150 mathematics books for teachers, and students aged 6–12.

Introduction

All students should be able to successfully calculate both mentally and with paper and pencil. The first significant step for calculating involves learning the basic number facts associated with each operation. The term "basic facts" refers to those facts that most people are expected to know automatically. The basic addition facts (and related subtraction facts) range from $0 + 0 = 0$ to $9 + 9 = 18$ inclusive. The basic multiplication facts (and related division facts) range from $0 \times 0 = 0$ to $9 \times 9 = 81$. These facts form the basis for all future number work.

Thinking strategies

The most effective way for students to learn the basic facts is to arrange the facts into clusters (Fuson 2003; Thornton, 1990). Each cluster is based on a thinking strategy that students can use to help them learn all of the facts in that cluster.

For example, the idea of using a double, or numbers close to a double, forms the basis of the strategy in the "use doubles" addition cluster. The other addition clusters involve "counting on" small numbers and "bridging to ten" when one addend is close to ten. Between these three clusters, students can master all 100 basic addition facts.

"Using addition" is the most effective thinking strategy for helping students to learn the basic subtraction facts. This is because subtraction is the inverse operation of addition. Both operations involve a part-part-total structure, but the known quantities vary. With addition the parts are known, but not the total; with subtraction the total and one part are known, but not the other part.

The idea of using a known multiplication fact involving ten, forms the basis of the strategy in the "use tens" multiplication cluster for the fives facts. The other multiplication clusters involve doubling for the twos, fours, and eights facts; using a rule to multiply by one or zero; and building up or down from a known fact for the sixes and nines facts. The majority of the remaining facts are covered by the turnarounds of the above.

As subtraction is the inverse operation of addition, division is the inverse operation of multiplication. Both multiplication and division involve an equal parts-total structure, but the known quantities vary. With multiplication, the known quantities are the number of equal parts and the number in each part, and the total is unknown. With division, the known quantities are the total, together with either the number of equal parts or the number in each of the equal parts. Because of this relationship between the two operations, "using multiplication" is the most effective thinking strategy for helping students to learn the basic division facts.

For subtraction and division, the strategies for the clusters of facts parallel those that are used for addition and multiplication. Where addition and multiplication have turnaround facts such as 6 + 2 = 8 and 2 + 6 = 8, or 6 × 4 = 24 and 4 × 6 = 24, subtraction and division have related or "partner" facts such as 8 – 6 = 2 and 8 – 2 = 6, or 24 ÷ 6 = 4 and 24 ÷ 4 = 6. Addition and subtraction facts that involve the same parts and total form "fact families"; multiplication and division facts with the same parts and total form "fact families" also.

Teaching sequence and stages

Mathematics is a discipline of connected ideas. Knowledge of a single concept or skill is often the foundation for many other aspects within the discipline. New ideas cannot be formed if the prerequisite concepts and skills are not well established. Research by learning theorists from the past thirty to forty years shows that for skills, it is essential that relevant practice occur frequently over time. For this reason, mathematics educators have long recommended that the curriculum be organized into a careful sequence that allows for development of understanding and mastery of skills.

The most widely accepted sequence for teaching the addition facts is as follows:

- Count on (1, 2, 3, and 0) with their turnarounds
- Use doubles (double, double plus 1, and double plus 2) with their turnarounds
- Bridge to 10 with their turnarounds

The sequence for teaching the multiplication facts is:

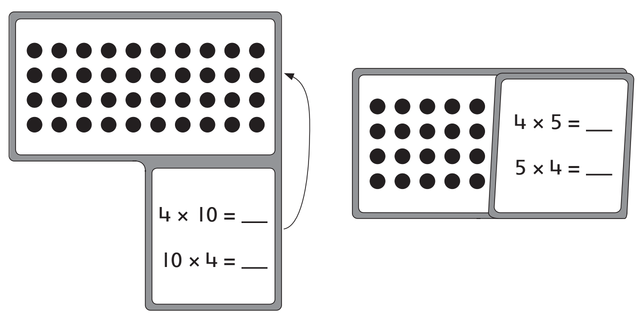
- Use tens (fives facts) with their turnarounds
- Doubling (twos, fours, and eights facts) with their turnarounds
- Use a rule (ones and zeros facts) with their turnarounds
- Build up and build down (sixes and nines facts) with their turnarounds

Along with the above sequences for teaching the strategies, the activities for each strategy are also sequenced according to the following four stages of teaching and learning: introduce the strategy, reinforce the strategy, practice the facts, extend the strategy. The use tens multiplication strategy for the fives facts will be used to explain and illustrate each of these stages.

Introduce the strategy

Hands-on materials, real-world stories, pictures, discussions, and familiar visual aids are typical of the activities that are used to model and introduce each thinking strategy in the first stage.

For multiplication, a rectangular array of dots is often used to show facts and their turnarounds, for example six rows of ten dots show 6 × 10 or 10 × 6. Upon seeing this, the student should be able to say that ten sixes are sixty. This numeration skill is pre-requisite to many other skills. A card such as the one below, can then be used to demonstrate the thinking required to figure out a fives fact. After determining the total number of dots, fold or cover up half the array, and have the student verbalize what they see and how they can figure out the total number of dots.

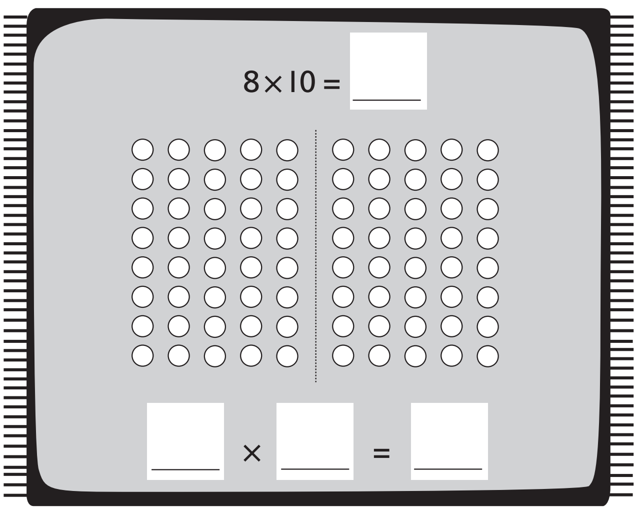


"I know ten fours are forty, so five fours must be half of that—twenty".

Reinforce the strategy

The activities in this stage are designed to make links between the concrete/pictorial and symbolic representations of the facts being examined. Students also reflect on how the strategy works and the numbers to which it applies.

Given pictures such as the one below, students can be asked to write the answer to a tens fact they see, then color half the array and write a number fact to match what is now colored. Experiences such as this serve to reinforce the relationship between the two facts and the thinking strategy that is involved.



Practice the facts

At this stage, games, flashcards, worksheets, and other activities provide students with opportunities to apply and demonstrate their knowledge of the facts. The students should use mental computation only and fast recall is stressed.

The following game for two or three players is a simple way to practice the fives facts.

Materials:

- About ten counters of one color for each student.
- Two blank cubes with the numerals 2, 4, 3, 3, 5, and 5 written on the faces of one cube and 6, 8, 7, 7, 9, and 9 on the faces of the other. Each 6 and 9 should be underlined to avoid confusion.
- A game board. Have the students sketch the game board below onto a grid or sheet of paper.

10	45	25	20
35	15	45	25
30	35	15	40

To play the game:

- The first player rolls both cubes.
- The player then chooses one of the numbers and multiplies it by five. Encourage the player to verbalize their thinking, for example, "Four tens are forty, so four fives are twenty."
- The player finds the product and places a counter on the game board.
- The next player has a turn.
- As the game continues, a player misses a turn if they cannot cover a number on the board (only one counter can be placed on any one space).
- The player who has the greatest number of counters on the board at the end is the winner.

As the students play the game, ask questions such as: *"What do you need to roll to place a counter on that number?"*

Extend the strategy

Once mastered, the strategies and skills can be applied to new contexts and situations. Students are encouraged to apply the strategy to numbers beyond the range of the basic number facts. The activities in this stage are designed to further strengthen students' number sense or "feel" for numbers.

To extend the use tens strategy, the students can explain how they can use the answer to the first number sentence of each pair below to answer the next number sentence in each pair.

$10 \times 18 =$ ____ so $5 \times 18 =$ ____

$10 \times 26 =$ ____ so $5 \times 26 =$ ____

$10 \times 22 =$ ____ so $5 \times 22 =$ ____

$10 \times 15 =$ ____ so $5 \times 15 =$ ____

Later, students will learn to vary this strategy by doubling and halving the two factors rather than doubling one factor and halving the product. This variation allows the strategy to work with number sentences other than those involving 5 as a factor. For example, by doubling and halving, students can quickly calculate that 14 × 35 has the same answer as 7 × 70, which is easy to calculate mentally.

Conclusion

It has long been known that students should have instant and accurate recall of the basic number facts. But now, more so than ever before, with the increased emphasis on mental computation in contemporary curricula, it is essential that these facts are taught using strategies that serve as the foundation of mental computation.

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