

# CREATING AHAS!

**Using Objects, Pictures, Strategies, and Symbols to  
Build Computational Fluency for All**

Sandra L. Atkins Ph.D.  
s\_atkins@origomath.com  
www.origoeducation.com

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## ***Creating Ahas!***

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For more information, contact

Phone 1-888-674-4601

Fax 1-888-674-4604

[sales@origomath.com](mailto:sales@origomath.com)

[www.origoeducation.com](http://www.origoeducation.com)

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1. Solve each of the problems below.

a. 
$$\begin{array}{r} 500 \\ - 327 \\ \hline \end{array}$$

b. 
$$\begin{array}{r} 305 \\ - 111 \\ \hline \end{array}$$

c. 
$$\begin{array}{r} 2001 \\ - \quad 3 \\ \hline \end{array}$$

d. 
$$\begin{array}{r} 199 \\ + 199 \\ \hline \end{array}$$

e. 
$$\begin{array}{r} 399 \\ + 57 \\ \hline \end{array}$$

f. 
$$\begin{array}{r} 2562 \\ + 3239 \\ \hline \end{array}$$

g.  $\frac{1}{4}$  of \$2.80

h. 15% of 420

2. List the strategy you used to solve each problem.

## Students who are computationally fluent ...

- select and use a variety of methods and tools to compute, including objects, mental computation, estimation, paper and pencil, and calculators.
- work flexibly with basic number combinations for adding, subtracting, multiplying, and dividing.
- use visual models, benchmarks, and equivalent forms when adding, subtracting, multiplying, and dividing whole numbers and fractions.

**GOAL:** Students are able to compute efficiently and accurately.

**NEED:** MEANINGFUL practice.

NCTM, *Principles and Standards for School Mathematics, 2000*

# ***Number Sense Components***

- ★ Count
- ★ Subitize
- ★ Relative Magnitude
- ★ Absolute Magnitude
- ★ Use Benchmarks
- ★ Compose and Decompose
- ★ Effects of Operations
- ★ Compute Flexibly

# ***The Bridge to Ten Strategy Pre-activities***


# The Bridge to Ten Strategy Pre-activities

## Activity 3 Dominoes—Total the Ends

### How to play:

- Challenge each student to select four dominoes at random and take turns to arrange the tiles as shown below, so the total number of dots on the outer ends is a multiple of five. The number made is that player's score. No points are scored if a multiple of five cannot be made.

Several scores are possible with the tiles above, but the greatest possible score is 20 as shown. The first player to reach 100 points wins the game.

### Variation

If one player cannot make a scoring arrangement with their tiles and another player can, the second player scores the relevant number of points.

The diagram shows four dominoes arranged in a cross shape. The top domino is 5-6, the bottom is 6-5, the left is 1-2, and the right is 1-1. The equation  $5 + 6 + 0 + 9 = 20$  is shown to the right of the bottom domino.

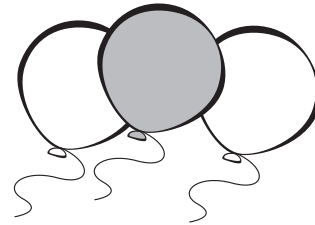
Adapted from 'Number Concept Activities' in *A Little Book of Big Ideas: Dominoes*, Calvin Irons (ORIGO Education, 2007).

# The Bridge to Ten Strategy Reinforce

WARM UP **11**

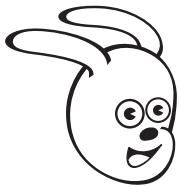
Name: \_\_\_\_\_

Jade had 9 girls and 6 boys at her party.  
How many guests in all?

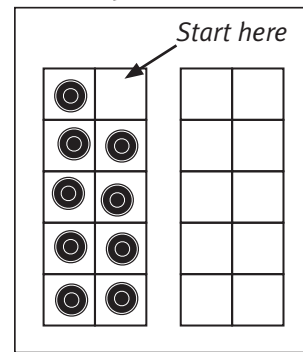


1. a. These ten-frames show 9 counters.  
Draw 6 more counters.

- b. Write the total. \_\_\_\_\_



What did you notice?

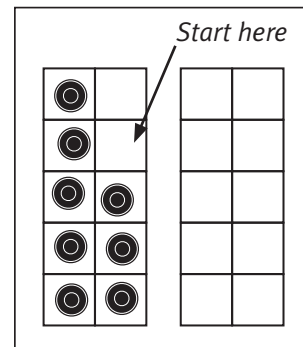


- c. Complete the sentence.

$9 + 6$  is the same as  $10 +$  \_\_\_\_\_

2. a. These ten-frames show 8.  
Draw 4 more counters.

- b. Write the total. \_\_\_\_\_



- c. Complete the sentence.

$8 + 4$  is the same as  $10 +$  \_\_\_\_\_

# The Bridge to Ten Strategy Reinforce

WORK OUT 11

Name: \_\_\_\_\_

1. Draw more counters then complete the sentence.

a. Draw 7 more

●			
●	●		
●	●		
●	●		
●	●		

9 + \_\_\_\_\_  
is the same as  
10 + \_\_\_\_\_ = \_\_\_\_\_

b. Draw 5 more

●			
●	●		
●	●		
●	●		
●	●		

9 + \_\_\_\_\_  
is the same as  
10 + \_\_\_\_\_ = \_\_\_\_\_

c. Draw 5 more

●			
●			
●	●		
●	●		
●	●		

\_\_\_\_\_ + \_\_\_\_\_  
is the same as  
\_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_

d. Draw 3 more

●			
●			
●	●		
●	●		
●	●		

\_\_\_\_\_ + \_\_\_\_\_  
is the same as  
\_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_

2. For each of these, draw an arrow to a number sentence below that has the same answer. Write the answer.

a.  $9 + 8$

b.  $8 + 6$

c.  $9 + 3$

$10 + 2 =$  \_\_\_\_\_

$10 + 7 =$  \_\_\_\_\_

$10 + 4 =$  \_\_\_\_\_

# The Bridge to Ten Strategy Extend

Name: \_\_\_\_\_

**WORK OUT** **12**

1. For each of these, draw more counters then complete the sentence.

a. Draw 7 more

$19 + \underline{\quad}$   
 is the same as  
 $20 + \underline{\quad} = \underline{\quad}$

b. Draw 6 more

$29 + \underline{\quad}$   
 is the same as  
 $\underline{\quad} + \underline{\quad} = \underline{\quad}$

c. Draw 5 more

$\underline{\quad} + \underline{\quad}$   
 is the same as  
 $\underline{\quad} + \underline{\quad} = \underline{\quad}$

d. Draw 8 more

$\underline{\quad} + \underline{\quad}$   
 is the same as  
 $\underline{\quad} + \underline{\quad} = \underline{\quad}$

2. For each of these, draw an arrow to a number sentence below that has the same answer. Write the answer.

a.  $29 + 8$     b.  $58 + 3$     c.  $28 + 7$     d.  $59 + 5$

$30 + 5 = \underline{\quad}$      $30 + 7 = \underline{\quad}$      $60 + 4 = \underline{\quad}$      $60 + 1 = \underline{\quad}$

# The Bridge to Ten Strategy Extend

Name: \_\_\_\_\_

WORK OUT **6**

1. For each of these, draw more counters then complete the sentence.

a. Draw 16 more

29 + \_\_\_\_\_ is the same as  
30 + \_\_\_\_\_ = \_\_\_\_\_

b. Draw 17 more

27 + \_\_\_\_\_ is the same as  
30 + \_\_\_\_\_ = \_\_\_\_\_

c. Draw 27 more

\_\_\_\_\_ + \_\_\_\_\_ is the same as  
\_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_

d. Draw 17 more

\_\_\_\_\_ + \_\_\_\_\_ is the same as  
\_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_

2. For each of these, draw an arrow to a number sentence below that has the same answer. Write the answer.

a.  $47 + 18$     b.  $68 + 27$     c.  $69 + 33$     d.  $48 + 26$

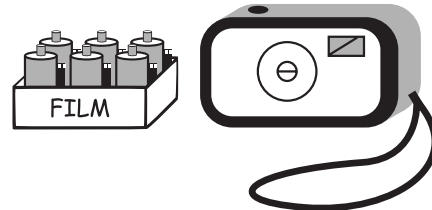
$50 + 24 =$  \_\_\_\_\_     $50 + 15 =$  \_\_\_\_\_     $70 + 25 =$  \_\_\_\_\_     $70 + 32 =$  \_\_\_\_\_

# The Bridge to Ten Strategy Extend

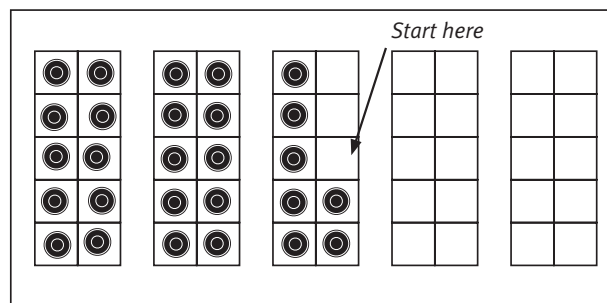
## WARM UP **6**

Name: \_\_\_\_\_

Aden bought a camera for \$27  
and a film pack for \$18.  
How much did he spend in all?



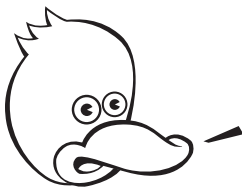
1. a. These ten-frames show 27 counters.  
Draw 18 more counters.



- b. Write the total. \_\_\_\_\_

- c. Complete this sentence to match the picture above.

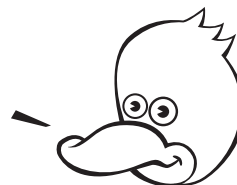
$27 + 18$  is the same as  $30 +$  \_\_\_\_\_



What do you notice?

2. Use the same method to figure out  $148 + 27$  in your head.

Try making a ten.



Complete this sentence to help you.

$148 + 27$  is the same as  $150 +$  \_\_\_\_\_

# The Bridge to Ten Strategy Extend

## WORK OUT 6

Name: \_\_\_\_\_

1. For each of these, draw an arrow to a number sentence below that has the same answer. Write the answer.

a.  $97 + 38$

b.  $128 + 47$

c.  $119 + 26$

$130 + 45 = \underline{\quad}$

$100 + 35 = \underline{\quad}$

$120 + 25 = \underline{\quad}$

2. Complete each sentence.

a.  $68 + 27$   
is the same as  
 $70 + \underline{\quad}$

b.  $247 + 36$   
is the same as  
 $250 + \underline{\quad}$

c.  $169 + 26$   
is the same as  
 $170 + \underline{\quad}$

d.  $38 + 127$   
is the same as  
 $\underline{\quad} + 130$

e.  $26 + 139$   
is the same as  
 $\underline{\quad} + 140$

f.  $47 + 329$   
is the same as  
 $\underline{\quad} + 330$

3. For each of these, draw an arrow to the number sentence you could use to figure it out. Write the answer.

a.  $138 + 47$

$120 + 65 = \underline{\quad}$

b.  $66 + 119$

$170 + 15 = \underline{\quad}$

c.  $28 + 157$

$140 + 45 = \underline{\quad}$

d.  $168 + 17$

$160 + 25 = \underline{\quad}$

# The Bridge to Ten Strategy Extend

Name: \_\_\_\_\_

**WORK OUT** **5**

1. For each of these, write an easier number sentence that will help you figure out the problem below. Write the answer.

a. 

_____ + _____ = _____
<b>SO</b>
$4.6 + 3.8 =$ _____

b. 

_____ + _____ = _____
<b>SO</b>
$8.9 + 4.3 =$ _____

c. 

_____ + _____ = _____
<b>SO</b>
$7.8 + 5.4 =$ _____

d. 

_____ + _____ = _____
<b>SO</b>
$1.9 + 6.5 =$ _____

e. 

_____ + _____ = _____
<b>SO</b>
$2.8 + 3.6 =$ _____

f. 

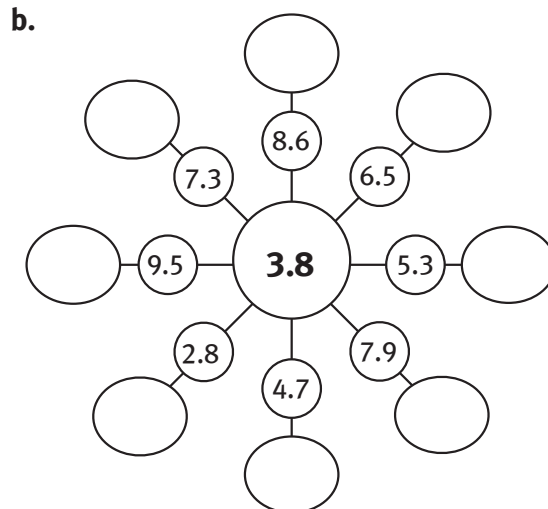
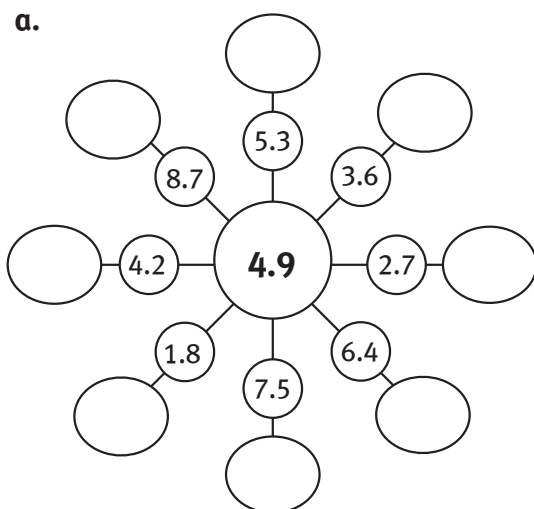
_____ + _____ = _____
<b>SO</b>
$9.9 + 6.7 =$ _____

2. Write the answers. Place a ✓ above the numbers you adjusted.

a.  $6.9 + 8.4 =$  \_\_\_\_\_      b.  $5.3 + 7.8 =$  \_\_\_\_\_      c.  $8.9 + 4.4 =$  \_\_\_\_\_

d.  $7.7 + 8.8 =$  \_\_\_\_\_      e.  $6.5 + 3.9 =$  \_\_\_\_\_      f.  $4.8 + 4.9 =$  \_\_\_\_\_

3. Add the numbers on the spokes to the number in the center. Write the answers around the outside.



# The Bridge to Ten Strategy Extend

Name: \_\_\_\_\_

**WORK OUT** **5**

1. Adjust each of these to make a new sentence that is easier to figure out. Write the answer.

a.  $23.9 + 15.6$   
is the same as  
\_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_

b.  $42.7 + 14.8$   
is the same as  
\_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_

c.  $31.8 + 16.9$   
is the same as  
\_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_

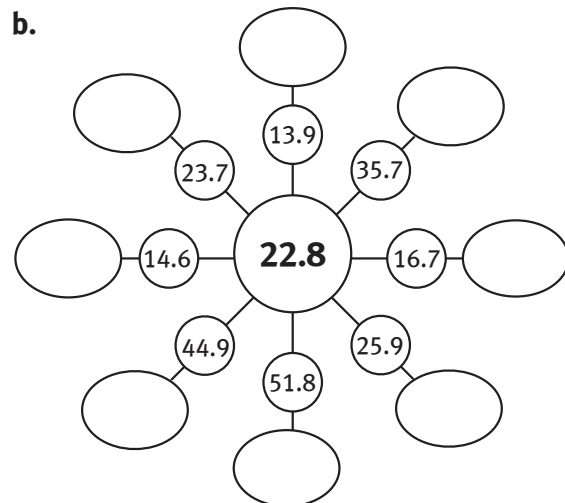
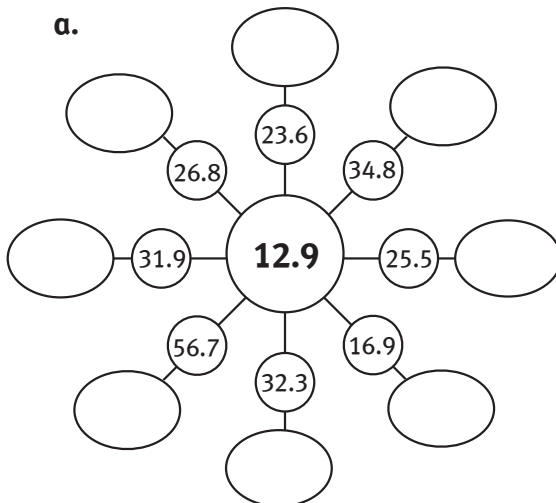
d.  $53.8 + 24.7$   
is the same as  
\_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_

2. Write the answers. Place a ✓ above the numbers you adjusted.

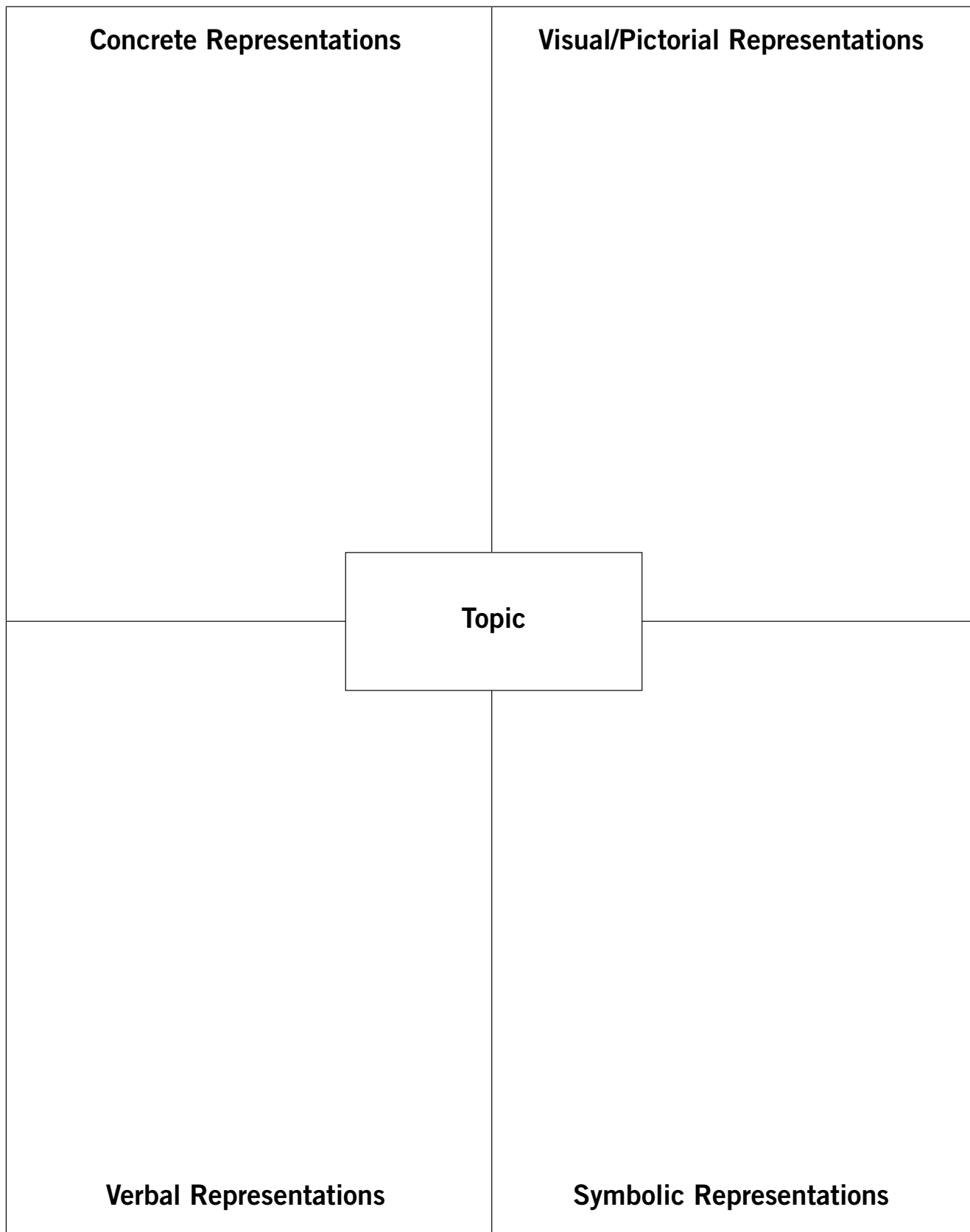
a.  $24.3 + 13.9 =$  \_\_\_\_\_    b.  $16.8 + 31.7 =$  \_\_\_\_\_    c.  $45.9 + 11.6 =$  \_\_\_\_\_

d.  $32.7 + 15.8 =$  \_\_\_\_\_    e.  $53.9 + 34.8 =$  \_\_\_\_\_    f.  $26.7 + 42.9 =$  \_\_\_\_\_

3. Add the numbers on the spokes to the number in the center.  
Write the answers around the outside.



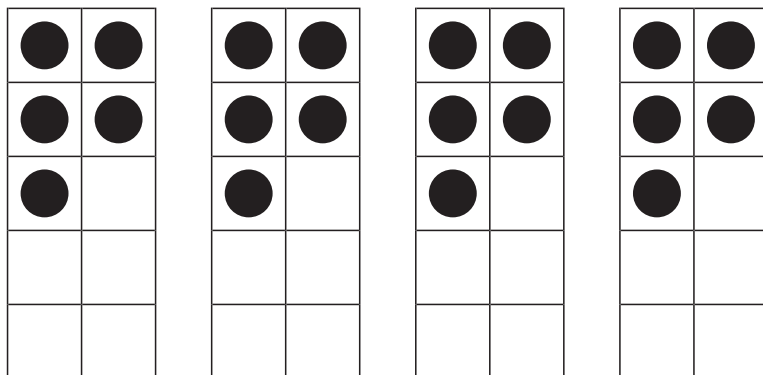
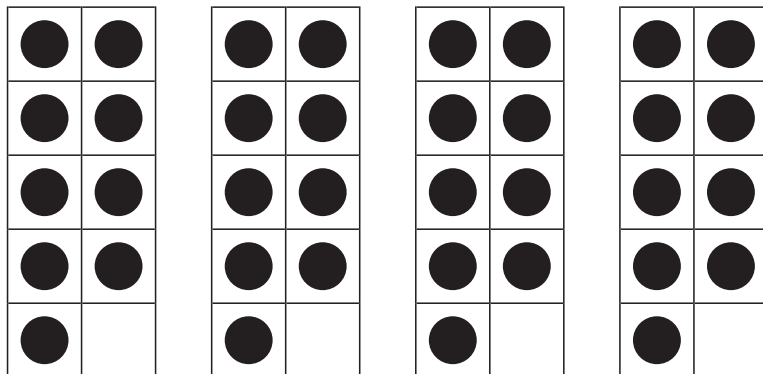
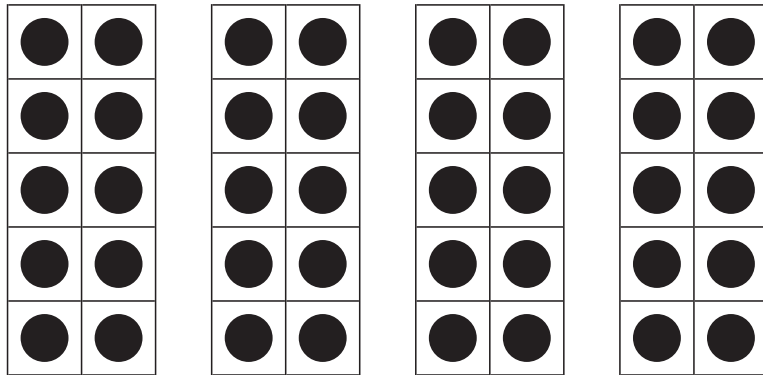
# ***Key Models for Building Connections***



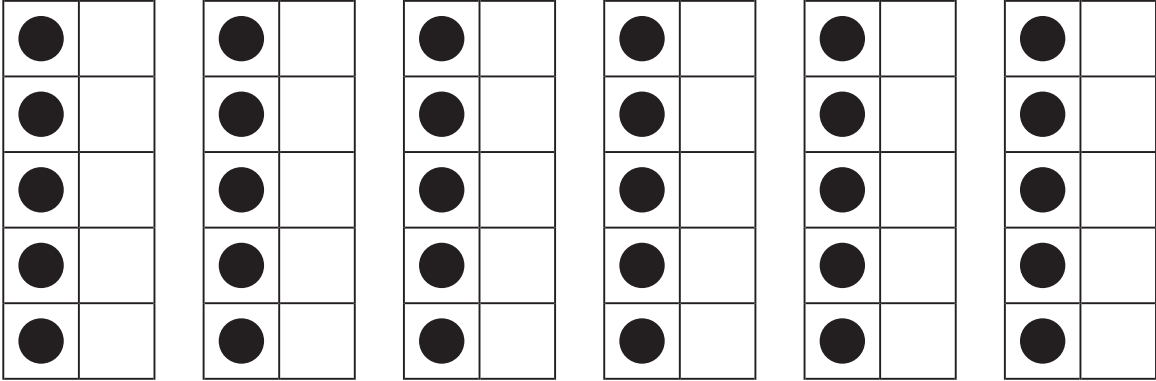
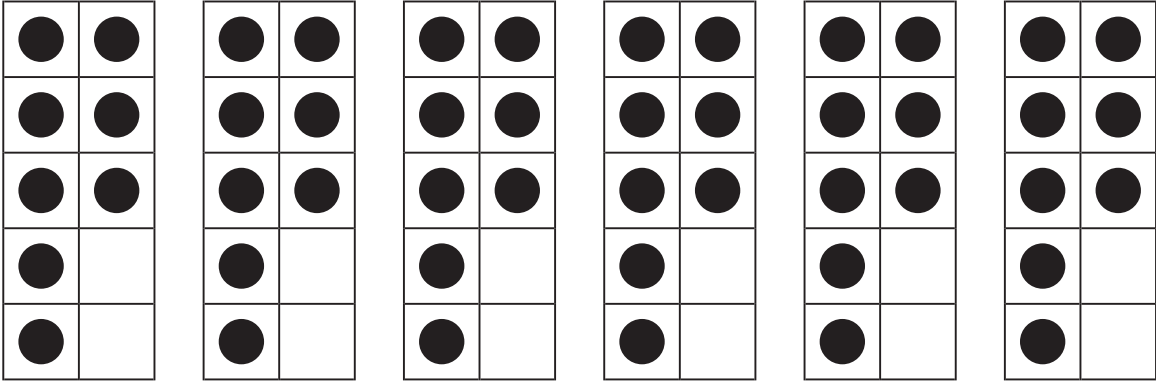
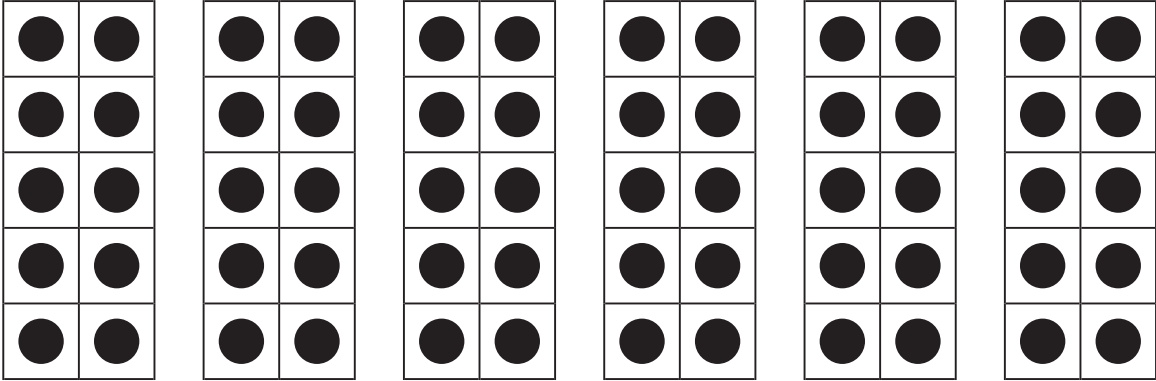
# Multiplication

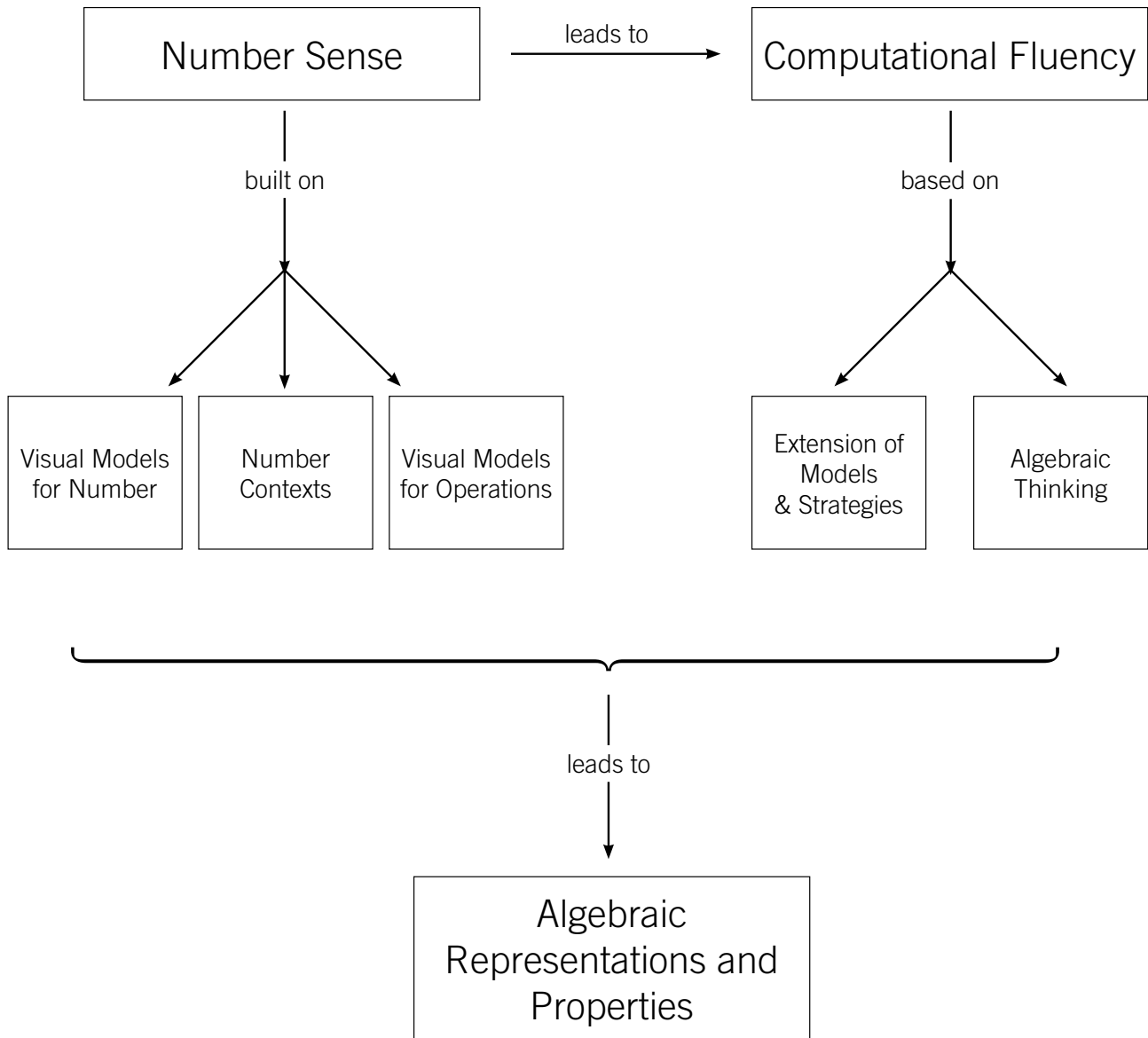
**Materials:** Copy the ten frames pictures below and on the next page onto overhead transparencies.

**Instructions:** Show each row of ten frames on the projector, one at a time by turning the projector on and off quickly. If needed show the picture again by quickly turning the projector on and off. Ask the students to tell the total and how they know.



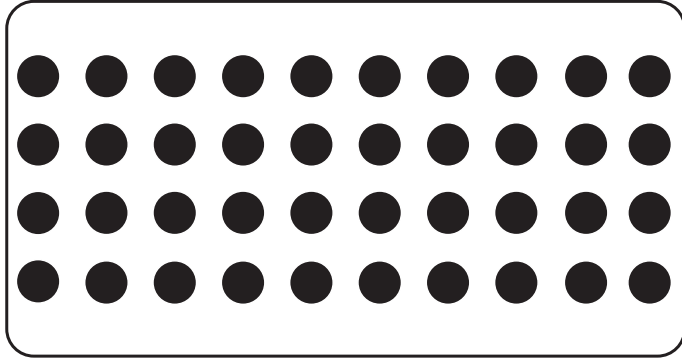
# Multiplication



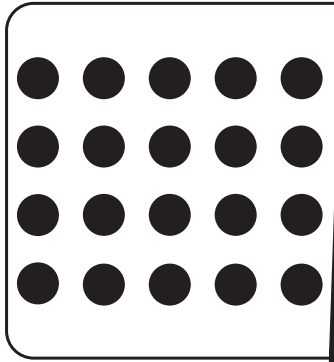


# ***Developing the number facts***

- Introduce
  - Use key visual models
- Reinforce
  - Link concrete, pictorial, verbal, and symbolic representations
- Practice
  - Build fluency through fun meaningful activities
- Extend
  - Apply to numbers beyond the basic number facts

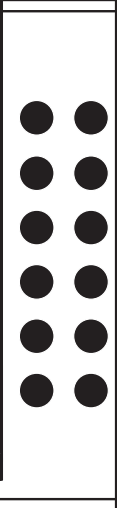


$4 \times 10 = \underline{\quad}$   
 $10 \times 4 = \underline{\quad}$



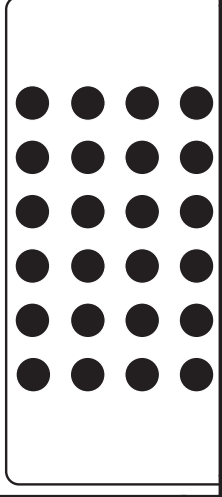
$4 \times 5 = \underline{\quad}$   
 $5 \times 4 = \underline{\quad}$

$2 \times 6 = \underline{\quad} = 6 \times 2$



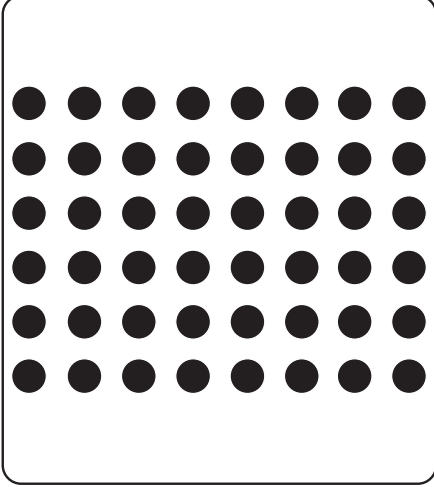
$2 \times 6 = \underline{\quad} = 6 \times 2$

$4 \times 6 = \underline{\quad} = 6 \times 4$



$2 \times 6 = \underline{\quad} = 6 \times 2$

$4 \times 6 = \underline{\quad} = 6 \times 4$



$8 \times 6 = \underline{\quad} = 6 \times 8$

# Sequencing multiplication fact experiences

Zeros facts

Ones facts

2s facts

4s facts

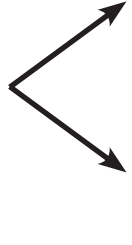
8s facts

10s facts

5s facts

6s facts

9s facts



# Nice and Easy

Using doubling and halving to multiply

2 or 3 players

## Purpose

In this game, the students are required to double and halve two factors to make a number sentence that is easier to calculate. In doing this, the students discover that the product remains unchanged. This is an efficient multiplication strategy when at least one factor in the equation is even.

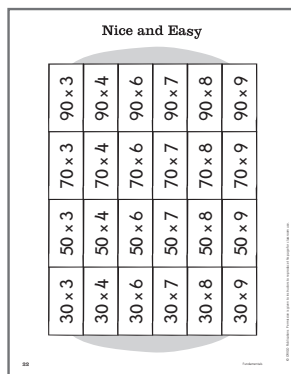
## Materials

Each group of players will need

- A 'Nice and Easy' game board (page 22) as shown below.
- Two (2) number cubes made from blank wooden cubes. One cube should show the numerals 15, 15, 25, 35, 45, and 45. The other cube should show 6, 8, 12, 14, 16, and 18.

Each player will need

- Fifteen (15) counters (a different color for each player).



The game board is a 4x4 grid of multiplication problems. The title 'Nice and Easy' is centered at the top. The problems are arranged in a 4x4 grid:

$30 \times 3$	$50 \times 3$	$70 \times 3$	$90 \times 3$
$30 \times 4$	$50 \times 4$	$70 \times 4$	$90 \times 4$
$30 \times 6$	$50 \times 6$	$70 \times 6$	$90 \times 6$
$30 \times 7$	$50 \times 7$	$70 \times 7$	$90 \times 7$
$30 \times 8$	$50 \times 8$	$70 \times 8$	$90 \times 8$
$30 \times 9$	$50 \times 9$	$70 \times 9$	$90 \times 9$

## How to Play

The aim is to arrange three counters adjacently in a horizontal, vertical, or diagonal line.

- The first player rolls the number cubes.
- The player says the multiplication sentence represented by the cubes, then doubles one factor and halves the other to figure out an equivalent sentence.

*Example: Lincoln rolls 35 and 16. He says, 35 times 16 is the same as 70 times 8.*

- The player states the product before claiming a corresponding space on the game board by covering it with a counter. If the space is unavailable, the player misses a turn.

*Example: Lincoln says, The answer is 560, and claims 70 x 8 on the game board.*

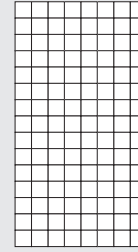
- The other player(s) has a turn.
- The first player to make a line of three adjacent counters is the winner.

## Reading the Research

There are different ways that students can be encouraged to move from inefficient to efficient thinking strategies. One technique is to talk about slightly more advanced procedures and why they work (Fuson & Kwon, 1992).

## Before the Game

Draw an array for  $15 \times 8$  on an overhead transparency as shown (right). Show the transparency and ask the students to write a number sentence to match what they see ( $15 \times 8$ ). Call upon a student to share his or her number sentence as you run your finger along the length and one side of the array to check. Ask the students if they can easily calculate the total number of squares in the array. If not, cut the array in half and rearrange the two pieces to make a new array as shown (right). Ask, *Have I changed the total number of squares in the array?* (No.) *What is the new number sentence you see now?* ( $30 \times 4$ .) *Is it easier to calculate the total number of squares now?* (Yes.) Make sure the students see that by cutting and rearranging the original array, one dimension (factor) is doubled and the other is halved.



$15 \times 8$



$30 \times 4$

## During the Game

Encourage the students to explain the strategy they use to find the answers. Different players will use different strategies. For example, after figuring out that  $35 \times 16$  is the same as  $70 \times 8$ , the following players gave these explanations:

**Lincoln:** *I knew 7 times 8 is 56 so 7 tens times 8 must be 56 tens or 560.*

**Letitia:** *I just doubled 70 then doubled and doubled again.*

**Lindsay:** *I knew 70 times 10 is 700, so 70 times 8 must be 140 less.*

## After the Game

Lead a discussion about the strategy introduced in the game. Ask, *Did you find this strategy easy (or difficult) to use? Why was it easy (or difficult)?*

Challenge the students to write some number sentences they think they can solve using a doubling and halving strategy. Do the students see that it does not work easily when both numbers are odd?

**Nice and Easy Too!**

90	150	210	270
120	200	280	360
180	300	420	540
210	350	490	630
240	400	560	720
270	450	630	810

## Beyond the Game

The students can play the same game using a different game board. Give them a copy of 'Nice and Easy Too!' on page 23 (illustrated). This time, the students say the equivalent multiplication sentence and cover the answer.

# Nice and Easy

$30 \times 3$	$50 \times 3$	$70 \times 3$	$90 \times 3$
$30 \times 4$	$50 \times 4$	$70 \times 4$	$90 \times 4$
$30 \times 6$	$50 \times 6$	$70 \times 6$	$90 \times 6$
$30 \times 7$	$50 \times 7$	$70 \times 7$	$90 \times 7$
$30 \times 8$	$50 \times 8$	$70 \times 8$	$90 \times 8$
$30 \times 9$	$50 \times 9$	$70 \times 9$	$90 \times 9$

# Nice and Easy Too!

90	150	210	270
120	200	280	360
180	300	420	540
210	350	490	630
240	400	560	720
270	450	630	810