

# Helping Struggling Students to Develop Computational Fluency by Connecting Key Math Models and Concepts

*Grades 3-5*

**PROFESSIONAL DEVELOPMENT MODULE**

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*Professional Development Module*

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1. Solve each of the problems below.

a. 
$$\begin{array}{r} 500 \\ - 327 \\ \hline \end{array}$$

b. 
$$\begin{array}{r} 305 \\ - 111 \\ \hline \end{array}$$

c. 
$$\begin{array}{r} 2001 \\ - 3 \\ \hline \end{array}$$

d. 
$$\begin{array}{r} 199 \\ + 199 \\ \hline \end{array}$$

e. 
$$\begin{array}{r} 399 \\ + 57 \\ \hline \end{array}$$

f. 
$$\begin{array}{r} 2562 \\ + 3239 \\ \hline \end{array}$$

g.  $\frac{1}{4}$  of \$2.80

h. 15% of 420

2. List the strategy you used to solve each problem.

# ***What mathematics do your students...***



struggle with?



don't understand?



can't do?



In column 1, record the misconceptions related to the previously identified struggles.

<b><i>Misconceptions</i></b> (What is it the students don't understand?)	<b><i>Needed Conceptual Understandings</i></b> (If a student really understood _____ they would understand...)

For each misconception listed in column 1, record the related conceptual understanding needed for students to be successful in mathematics.

# ***An overview of the process— Identifying the problem***

- Identify common problem areas
- Pinpoint the disconnect
- Identify possible causes—prior experiences that led to the disconnect
- Identify the misconception
- Identify the needed conceptual understanding
- Identify key experiences to develop the needed conceptual understanding

# ***What do you think?***

A student was solving some simple math fact problems in her mathematics journal. This is what she wrote.

$$4 + 5 = 5$$

$$4 + 6 = 6$$

$$4 + 7 = 7$$

$$4 + 8 = 8$$

$$4 + 9 = 9$$

What is the student's error?

What is the disconnect?

How would you intervene?

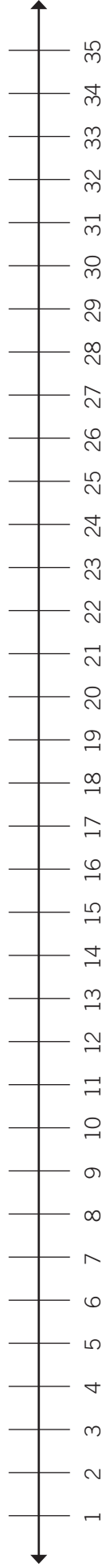
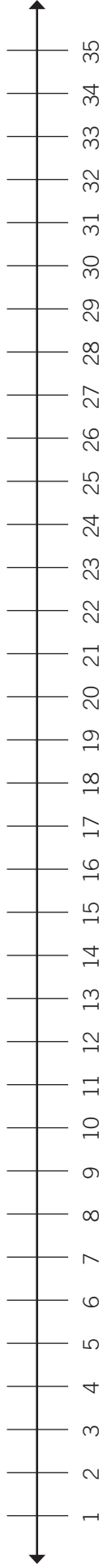
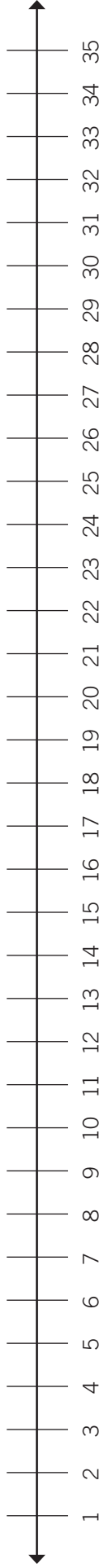
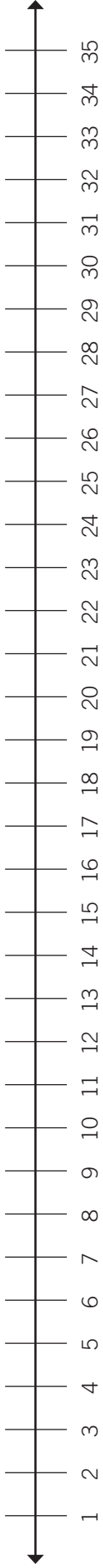
# ***Becoming a Detective— The art of researching the problem***

- Ask questions to find out what they are really thinking.
- Listen to what they say instead of what you hope to hear.
- Question to determine depth of understanding.
- Investigate before you correct.
- Make sure you are correcting the actual problem.
- Build on what they do understand.
- Don't correct too soon.

Remember the value of informal assessment...

*Formal assessments tell you there is a problem.*

*Informal assessments help determine what the problem is.*



# Key models for building connections the number line

## Up or Down

Adjusting numbers to subtract

2 or 3 players

### Purpose

In this game, the students are required to find the difference between two two-digit numbers. The students discover that changing both totals in the same way keeps the difference unchanged. This is an efficient mental strategy for subtraction, when the calculation involves bridging across a ten.

### Materials

Each group of players will need

- An 'Up or Down' game board (page 34) as shown below.
- Two (2) number cubes made from blank wooden cubes. One cube should show the numerals 92, 82, 72, 62, 52, and 42. The other cube should show 17, 27, 37, 17, 27, and 37.

Each player will need

- Fifteen (15) counters (a different color for each player).

90-15	95-20	90-25	95-30	90-35
80-15	85-20	80-25	85-30	80-35
70-15	75-20	70-25	75-30	70-35
60-15	65-20	60-25	65-30	60-35
50-15	55-20	50-25	55-30	50-35
40-15	45-20	40-25	45-30	40-35

### How to Play

The aim is to arrange three counters adjacently in a horizontal, vertical, or diagonal line.

- The first player rolls the number cubes.
- The player says the subtraction sentence represented by the cubes, then figures out and says an equivalent subtraction sentence.

*Example: Reece rolls 42 and 17, and says, 42 take away 17 is the same as 40 take away 15 or 45 take away 20.*

- The player states the difference before claiming a corresponding space on the game board by covering it with a counter. If both choices are unavailable, the player misses a turn.

*Example: Reece says, The answer is 25, and claims 40 - 15 or 45 - 20 on the game board.*

- The other player(s) has a turn.
- The first player to make a line of three adjacent counters is the winner.

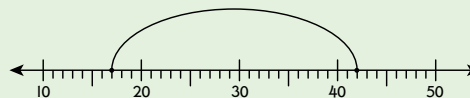
### Reading the Research

Research shows that children need many different kinds of number experiences to help them deal with real-life situations. It is important that they work regularly with a variety of number representations, including a counting model, a linear model such as a number line, a place-value model and a quantity model (Irons, 2002).

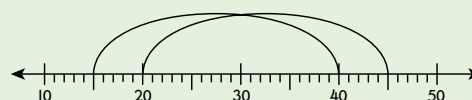
# Key models for building connections the number line

## Before the Game

Draw a number line like that shown (right) on an overhead transparency. Invite a volunteer to roll the number cubes and mark the location of the numbers on the line. This number line shows that 42 and 17 was rolled. Place another blank transparency over the number line. Draw a loop to show the difference between the two numbers. Say, *If we can't quickly calculate the difference, it is sometimes easier to change the two numbers.*



Slide the loop along the number line until the ends are resting on 40 and 15, and ask, *Have I changed the loop or difference?* (No.) *Is it easier to calculate  $40 - 15$  in our heads?* (Yes.) Slide the loop in the other direction until the ends are resting on 45 and 20. Again ask, *Is it easier to calculate  $45 - 20$ ?* (Yes.)



## During the Game

Encourage the students to explain the strategy they use to find the answers. Different players will use different strategies. For example, after figuring out that  $72 - 37$  is the same as  $70 - 35$ , the following players gave these explanations:

**Grace:** *I subtracted 30 then another 5. The answer is 35.*

**Grant:** *I just knew that double 35 is 70, so 70 take away 35 must be 35.*

**Gabby:** *I started with 35 and added 5 to get 40 and another 30 to get 70. The difference is  $5 + 30$ .*

## After the Game

Lead a discussion about the strategy introduced in the game. Ask, *Did you find this strategy easy (or difficult) to use? Why was it easy (or difficult)? Did you prefer to adjust the numbers down or up? Why?*

## Beyond the Game

- The students can play 'Up or Down' using a different pair of cubes. On the faces of one cube, write the numerals 18, 28, 38, 18, 28, and 38. The other cube should show 93, 83, 73, 63, 53, and 43.
- The students can play the same game using a different game board. Give them a copy of 'Up or Down Again' shown on page 35 (illustrated). This time, the students verbalize the equivalent number sentence and cover the answer.

Up or Down Again

75	65	55	45	35	25	15	5
65	55	45	35	25	15	5	
55	45	35	25	15	5		
45	35	25	15	5			
35	25	15	5				
25	15	5					

# Key models for building connections the number line

## Up or Down

90 - 15	95 - 20	90 - 25	95 - 30	90 - 35
80 - 15	85 - 20	80 - 25	85 - 30	80 - 35
70 - 15	75 - 20	70 - 25	75 - 30	70 - 35
60 - 15	65 - 20	60 - 25	65 - 30	60 - 35
50 - 15	55 - 20	50 - 25	55 - 30	50 - 35
40 - 15	45 - 20	40 - 25	45 - 30	40 - 35

# Key models for building connections the number line

## Up or Down Again

75	75	65	65	55
65	65	55	55	45
55	55	45	45	35
45	45	35	35	25
35	35	25	25	15
25	25	15	15	5

# **Key models for building connections the number line**

1. 
$$\begin{array}{r} 600 \\ - 432 \\ \hline \end{array}$$

2. 
$$\begin{array}{r} 401 \\ - 134 \\ \hline \end{array}$$

3. 
$$\begin{array}{r} 2003 \\ - 867 \\ \hline \end{array}$$

4. 
$$\begin{array}{r} 332 \\ - 198 \\ \hline \end{array}$$

5. 
$$\begin{array}{r} 6.00 \\ - 4.32 \\ \hline \end{array}$$

6. 
$$\begin{array}{r} 3.01 \\ - 1.57 \\ \hline \end{array}$$

# Key models for building connections the number line

11

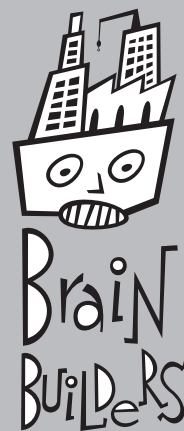
Write the answers that you can figure out **in your head**.

- a.  $80 - 29 = \underline{\quad}$
- b.  $400 - 299 = \underline{\quad}$
- c.  $5,604 - 600 = \underline{\quad}$
- d.  $165 - 75 = \underline{\quad}$
- e.  $35 - 21 = \underline{\quad}$

Computation and Number Sense

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6

Look at this strategy.

$$13.4 - 5.9 \text{ is the same as } 13.5 - 6$$

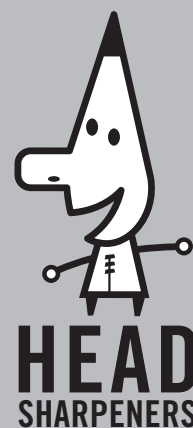
Copy and complete these. Try using the same strategy.

- a.  $23.4 - 6.9 = \underline{\quad}$
- b.  $15.3 - 2.8 = \underline{\quad}$
- c.  $19.2 - 4.7 = \underline{\quad}$
- d.  $16.6 - 7.8 = \underline{\quad}$

Computation and Number Sense

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# Key models for building connections the number line

14

- a. Figure out the answer to this number sentence  
**in your head.**

$$3,200 - 1,900 = \underline{\hspace{2cm}}$$

- b. Write about the strategy you used.  
c. Write **four** other number sentences that you could  
solve the same way.



Computation and Number Sense

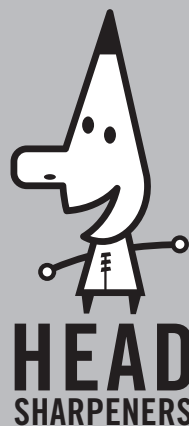
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12

Write the answers that you can figure out **in your head.**

- a.  $\$20 - \$15.25 = \underline{\hspace{2cm}}$   
b.  $36 - 7.8 = \underline{\hspace{2cm}}$   
c.  $8.97 - 6.93 = \underline{\hspace{2cm}}$   
d.  $\frac{3}{4} - \frac{1}{2} = \underline{\hspace{2cm}}$   
e.  $350 - 20 - 0.8 = \underline{\hspace{2cm}}$

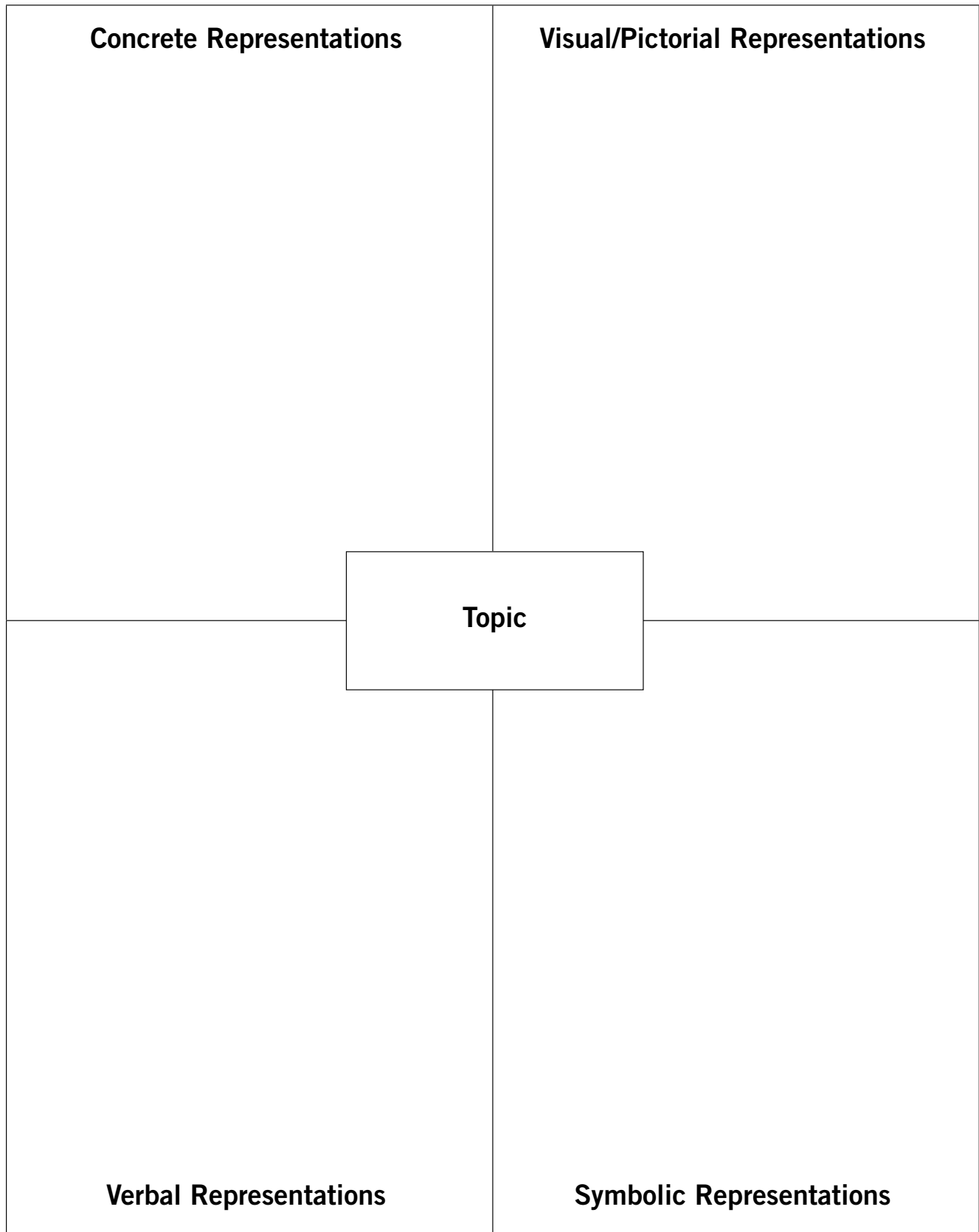


Computation and Number Sense

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# ***Key models for building connections the number line***



# **The importance of number sense**

## **A look at equivalencies**

**Find all of the ways to make 3.42.**

Tens	Ones	Tenths	Hundredths	Thousandths

**Find all of the ways to make  $2\frac{2}{3}$ .**

Ones	Halves	Thirds	Fourths	Sixths

**Find all of the ways to make \$2.63.**

Dollars	Quarters	Dimes	Nickels	Pennies

# ***The importance of number sense***

## ***A look at equivalencies***

**Find all of the ways to make 4365.**

Thousands	Hundreds	Tens	Ones

**Find all of the ways to make 62.**

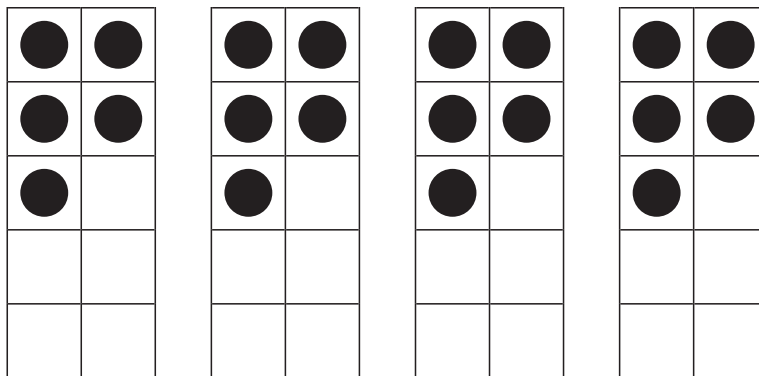
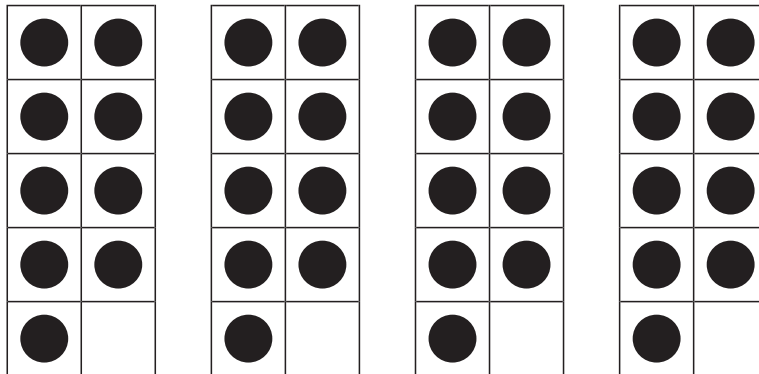
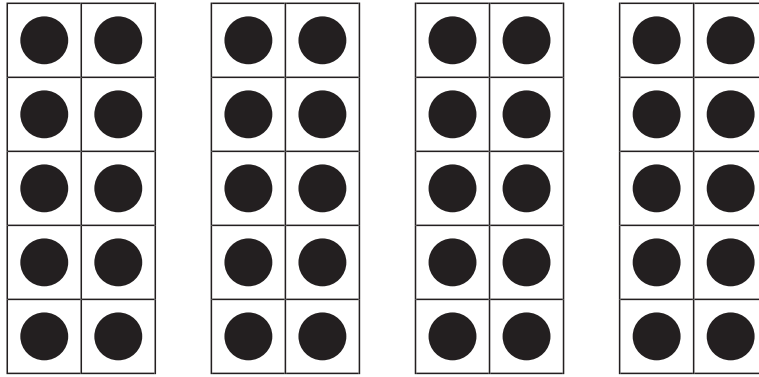
Tens	Ones

**Find all of the ways to make 10.**

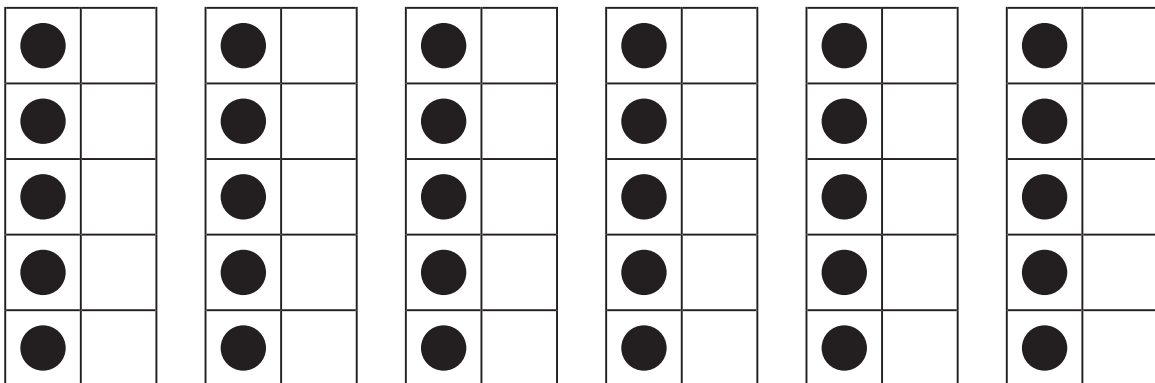
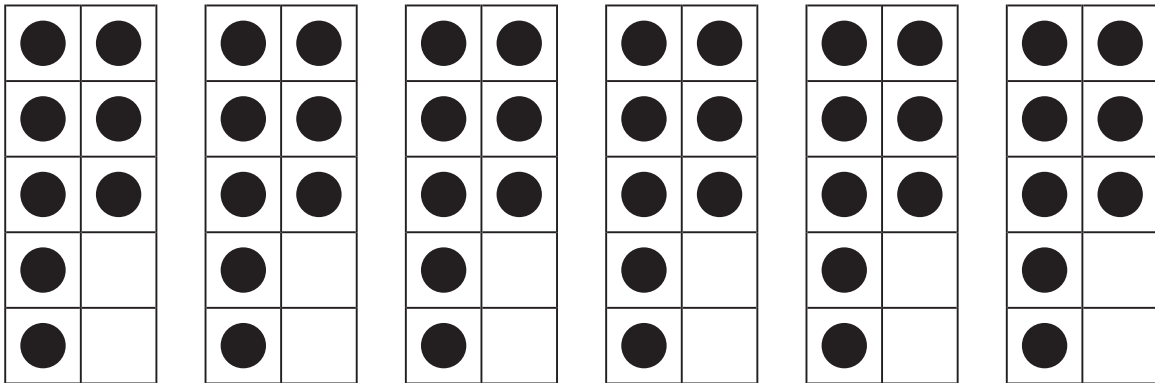
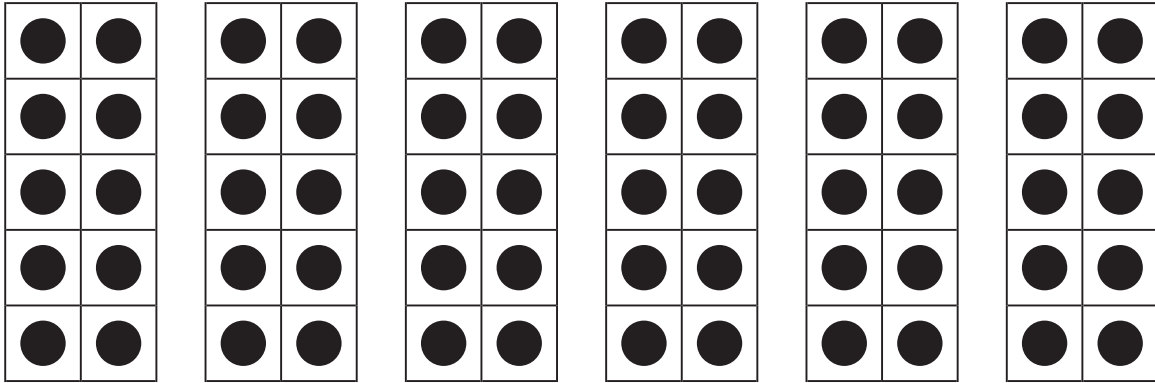
# Multiplication

**Materials:** Copy the ten frames pictures below and on the next page onto overhead transparencies.

**Instructions:** Show each row of ten frames on the projector, one at a time by turning the projector on and off quickly. If needed show the picture again by quickly turning the projector on and off. Ask the students to tell the total and how they know.

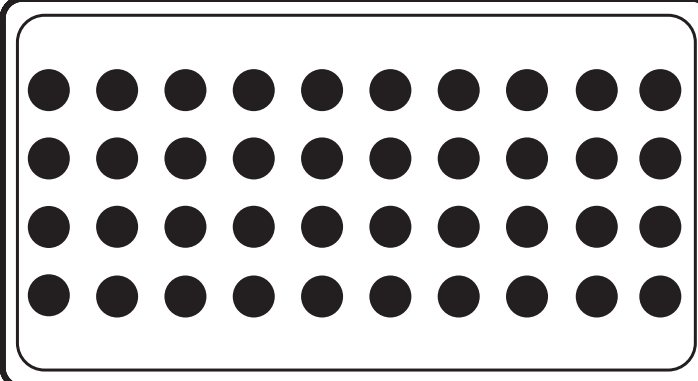


# Multiplication

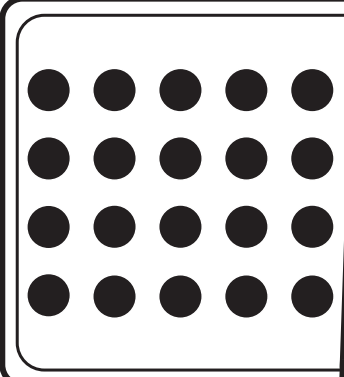


# Multiplication Chart

<b>X</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>
<b>0</b>	0	0	0	0	0	0	0	0	0	0
<b>1</b>	0	1	2	3	4	5	6	7	8	9
<b>2</b>	0	2	4	6	8	10	12	14	16	18
<b>3</b>	0	3	6	9	12	15	18	21	24	27
<b>4</b>	0	4	8	12	16	20	24	28	32	36
<b>5</b>	0	5	10	15	20	25	30	35	40	45
<b>6</b>	0	6	12	18	24	30	36	42	48	54
<b>7</b>	0	7	14	21	28	35	42	49	56	63
<b>8</b>	0	8	16	24	32	40	48	56	64	72
<b>9</b>	0	9	18	27	36	45	54	63	72	81




$4 \times 10 = \underline{\quad}$   
 $10 \times 4 = \underline{\quad}$



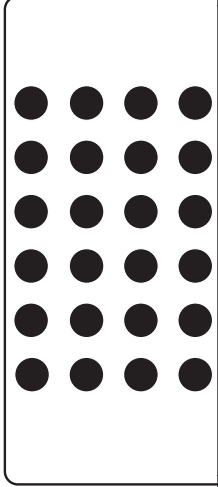
$4 \times 5 = \underline{\quad}$   
 $5 \times 4 = \underline{\quad}$

$2 \times 6 = \underline{\quad} = 6 \times 2$



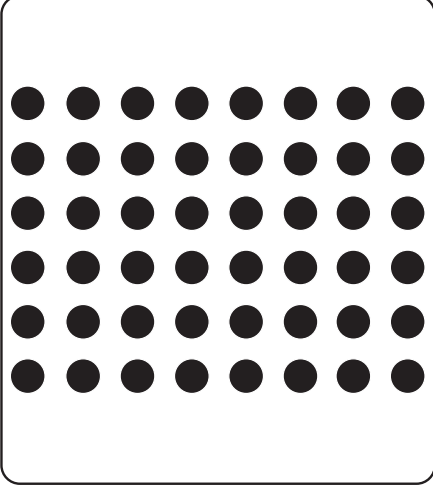
$2 \times 6 = \underline{\quad} = 6 \times 2$

$4 \times 6 = \underline{\quad} = 6 \times 4$



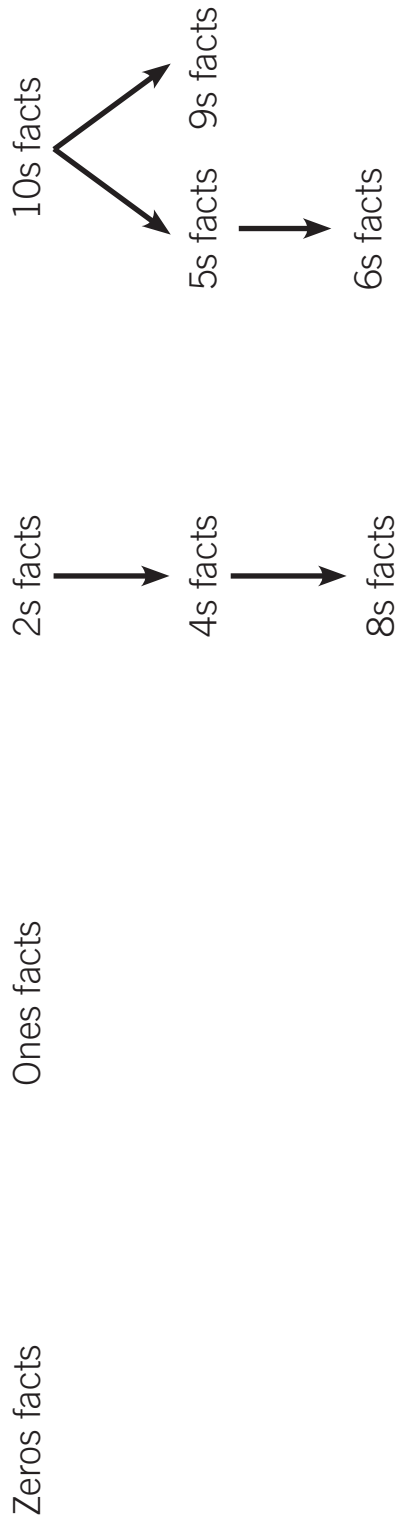
$2 \times 6 = \underline{\quad} = 6 \times 2$

$4 \times 6 = \underline{\quad} = 6 \times 4$



$8 \times 6 = \underline{\quad} = 6 \times 8$

# Sequencing multiplication fact experiences



# ***Summarizing the process***

## Planning for Intervention

- Identify common misconceptions
- Identify needed conceptual understandings
- Determine how far back to plug the hole
- Identify key visual models
- Link concrete, visual/pictorial, verbal, and symbolic representations
- Plan to bridge to grade level using similar models
- Be purposeful in material use and model selection
- Spiral

## Classroom Protocols

- Allow for mistakes—Don't panic!
- Ask versus tell (whenever possible)
- Use Think-pair-share and other informal interview techniques to determine understanding
- Ask questions to determine understandings versus reaching an instructional goal
- Require students to always explain thinking
- Build connections