

# Helping Struggling Students Develop Computational Fluency By Connecting Key Math Models And Concepts

**Grades 3-5**

Sandra L. Atkins Ph.D.  
s\_atkins@origomath.com

*NCTM Annual Meeting  
Washington DC  
April 2009*

***Helping Struggling Students Develop Computational Fluency  
By Connecting Key Math Models And Concepts***

Copyright 2009 ORIGO Education

For more information, contact

**North America**

Tel. 1-888-ORIGO-01

Fax 888-674-4604

[sales@origomath.com](mailto:sales@origomath.com)

[www.origomath.com](http://www.origomath.com)

All rights reserved. Unless specifically stated, no part of this publication may be reproduced, or copied into, or stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of ORIGO Education. No resale of this material is permitted.

# ***What mathematics do your students...***



struggle with?



don't understand?



can't do?



In column 1, record the misconceptions related to the previously identified struggles.

<b><i>Misconceptions</i></b> (What is it the students don't understand?)	<b><i>Needed Conceptual Understandings</i></b> (If a student really understood _____ they would understand...)

For each misconception listed in column 1, record the related conceptual understanding needed for students to be successful in mathematics.

# ***An overview of the process— Identifying the problem***

- Identify common problem areas
- Pinpoint the disconnect
- Identify possible causes—prior experiences that led to the disconnect
- Identify the misconception
- Identify the needed conceptual understanding
- Identify key experiences to develop the needed conceptual understanding

# ***The Bridge to Ten Strategy— Pre-activities***

## **Activity 1    Dominoes—A different look**

### **Materials:**

One set of double-nine five frame dominoes per group of 3 or 4

### **How to play:**

- Place all of the dominoes upside-down on the table.
- Each player selects 7 dominoes.
- The person with the largest double begins by placed the double on the table.
- Continuing clockwise, each player takes a turn matching their domino to one end of the domino train.
- If a player is unable to “attach” a domino to the train, he or she must draw from the pile until they can.
- The first player to play all of his or her dominoes wins.

# ***The Bridge to Ten Strategy— Pre-activities***

## **Activity 2    Dominoes—Who has more?**

### **Materials:**

One set of double-nine five frame dominoes per pair

### **How to play:**

- Turn all of the dominoes upside-down.
- Distribute all of the dominoes so that each player has half of the set.
- Each player places all of the dominoes in their hand upside-down in a stack on the table.
- Each player turns over the top card in the deck.
- The player with the greatest total describes how she knows she her domino has a greater total than her opponent's. That player takes both dominoes and places them at the bottom of the deck.
- If there is a tie, each player places 2 additional dominoes face down on his or her initial play. They then place a third domino face up on top of the stack. The player with the greatest total claims both stacks by describing how they determined they had the greatest total.
- Continue play until a player has captured all of the dominoes.

# ***The Bridge to Ten Strategy Pre-activities***


# The Bridge to Ten Strategy Pre-activities

## Activity 3 Dominoes—Total the Ends

### How to play:

- Challenge each student to select four dominoes at random and take turns to arrange the tiles as shown below, so the total number of dots on the outer ends is a multiple of five. The number made is that player's score. No points are scored if a multiple of five cannot be made.

Several scores are possible with the tiles above, but the greatest possible score is 20 as shown. The first player to reach 100 points wins the game.

### Variation

If one player cannot make a scoring arrangement with their tiles and another player can, the second player scores the relevant number of points.

5 + 6 + 0 + 9 = 20

Adapted from 'Number Concept Activities' in *A Little Book of Big Ideas: Dominoes*, Calvin Irons (ORIGO Education, 2007).

# ***The Bridge to Ten Strategy*** ***Introduce***



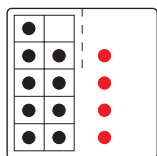
# The Bridge to Ten Strategy Introduce

The bridge-to-ten strategy can be used to introduce and develop these facts.

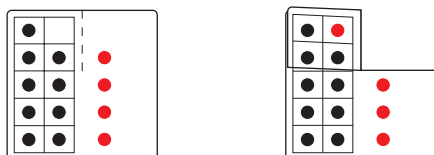
+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

## Directions

1. Hold one card as shown below. Ask the students to tell you what they see and how they can figure out the total number of dots.



2. Close the flap to demonstrate the idea of bridging to ten.



*Nine add four is the same as ten add three.*

3. Ask the students to write the two number facts to help them see that the answers are the same.

$$9 + 4 = 13$$

$$10 + 3 = 13$$

4. Repeat Steps 1 to 3 with other cards from the set.
5. Use the same cards to develop the turnaround facts. The addend that is closer to ten should be represented on the right.

# The Bridge to Ten Strategy Reinforce

WORK OUT 11

Name: \_\_\_\_\_

1. Draw more counters then complete the sentence.

a. Draw 7 more

\_\_\_\_\_ + \_\_\_\_\_  
is the same as  
10 + \_\_\_\_\_ = \_\_\_\_\_

b. Draw 5 more

\_\_\_\_\_ + \_\_\_\_\_  
is the same as  
10 + \_\_\_\_\_ = \_\_\_\_\_

c. Draw 5 more

\_\_\_\_\_ + \_\_\_\_\_  
is the same as  
\_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_

d. Draw 3 more

\_\_\_\_\_ + \_\_\_\_\_  
is the same as  
\_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_

2. For each of these, draw an arrow to a number sentence below that has the same answer. Write the answer.

a.  $9 + 8$

b.  $8 + 6$

c.  $9 + 3$

$10 + 2 =$  \_\_\_\_\_

$10 + 7 =$  \_\_\_\_\_

$10 + 4 =$  \_\_\_\_\_

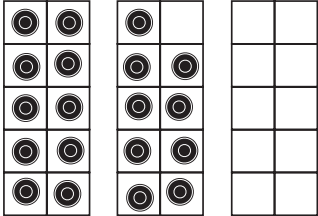
# The Bridge to Ten Strategy Extend

Name: \_\_\_\_\_

**WORK OUT** **12**

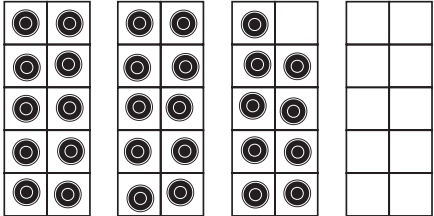
1. For each of these, draw more counters then complete the sentence.

a. Draw 7 more



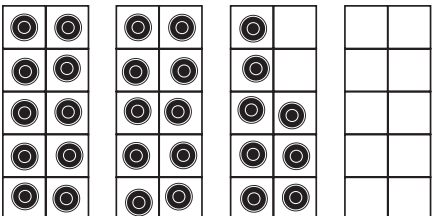
$19 + \underline{\quad}$   
 is the same as  
 $20 + \underline{\quad} = \underline{\quad}$

b. Draw 6 more



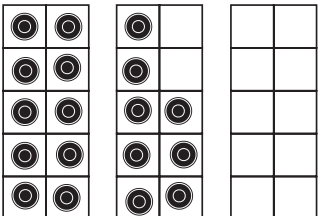
$29 + \underline{\quad}$   
 is the same as  
 $\underline{\quad} + \underline{\quad} = \underline{\quad}$

c. Draw 5 more



$\underline{\quad} + \underline{\quad}$   
 is the same as  
 $\underline{\quad} + \underline{\quad} = \underline{\quad}$

d. Draw 8 more



$\underline{\quad} + \underline{\quad}$   
 is the same as  
 $\underline{\quad} + \underline{\quad} = \underline{\quad}$

2. For each of these, draw an arrow to a number sentence below that has the same answer. Write the answer.

a.  $29 + 8$     b.  $58 + 3$     c.  $28 + 7$     d.  $59 + 5$

$30 + 5 = \underline{\quad}$      $30 + 7 = \underline{\quad}$      $60 + 4 = \underline{\quad}$      $60 + 1 = \underline{\quad}$

# The Bridge to Ten Strategy Extend

Name: \_\_\_\_\_

WORK OUT **6**

1. For each of these, draw more counters then complete the sentence.

a. Draw 16 more

●	●	●	●				
●	●	●	●				
●	●	●	●				
●	●	●	●				
●	●	●	●				

29 + \_\_\_\_\_ is the same as  
30 + \_\_\_\_\_ = \_\_\_\_\_

b. Draw 17 more

●	●	●	●				
●	●	●	●				
●	●	●	●				
●	●	●	●				
●	●	●	●				

27 + \_\_\_\_\_ is the same as  
30 + \_\_\_\_\_ = \_\_\_\_\_

c. Draw 27 more

●	●	●					
●	●	●					
●	●	●					
●	●	●					
●	●	●					

\_\_\_\_\_ + \_\_\_\_\_ is the same as  
\_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_

d. Draw 17 more

●	●	●					
●	●	●					
●	●	●					
●	●	●					
●	●	●					

\_\_\_\_\_ + \_\_\_\_\_ is the same as  
\_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_

2. For each of these, draw an arrow to a number sentence below that has the same answer. Write the answer.

a.  $47 + 18$     b.  $68 + 27$     c.  $69 + 33$     d.  $48 + 26$

$50 + 24 =$  \_\_\_\_\_     $50 + 15 =$  \_\_\_\_\_     $70 + 25 =$  \_\_\_\_\_     $70 + 32 =$  \_\_\_\_\_

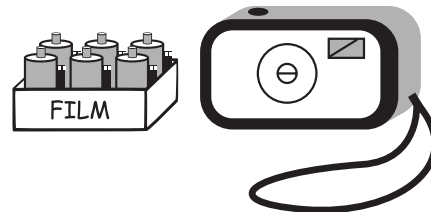
© ORIGO Publications. Permission is given to instructors to reproduce this page for classroom use.

# The Bridge to Ten Strategy Extend

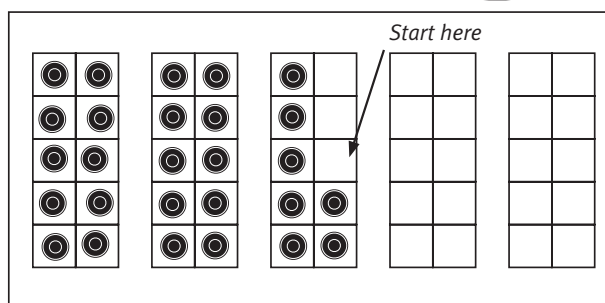
## WARM UP **6**

Name: \_\_\_\_\_

Aden bought a camera for \$27  
and a film pack for \$18.  
How much did he spend in all?



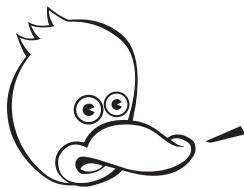
1. a. These ten-frames show 27 counters.  
Draw 18 more counters.



- b. Write the total. \_\_\_\_\_

- c. Complete this sentence to match the picture above.

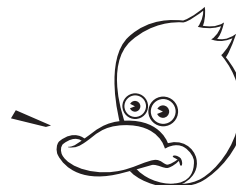
$27 + 18$  is the same as  $30 +$  \_\_\_\_\_



What do you notice?

2. Use the same method to figure out  $148 + 27$  in your head.

Try making a ten.



Complete this sentence to help you.

$148 + 27$  is the same as  $150 +$  \_\_\_\_\_

# The Bridge to Ten Strategy Extend

## WORK OUT 6

Name: \_\_\_\_\_

1. For each of these, draw an arrow to a number sentence below that has the same answer. Write the answer.

a.  $97 + 38$

b.  $128 + 47$

c.  $119 + 26$

$130 + 45 = \underline{\quad}$

$100 + 35 = \underline{\quad}$

$120 + 25 = \underline{\quad}$

2. Complete each sentence.

a.  $68 + 27$   
is the same as  
 $70 + \underline{\quad}$

b.  $247 + 36$   
is the same as  
 $250 + \underline{\quad}$

c.  $169 + 26$   
is the same as  
 $170 + \underline{\quad}$

d.  $38 + 127$   
is the same as  
 $\underline{\quad} + 130$

e.  $26 + 139$   
is the same as  
 $\underline{\quad} + 140$

f.  $47 + 329$   
is the same as  
 $\underline{\quad} + 330$

3. For each of these, draw an arrow to the number sentence you could use to figure it out. Write the answer.

a.  $138 + 47$

$120 + 65 = \underline{\quad}$

b.  $66 + 119$

$170 + 15 = \underline{\quad}$

c.  $28 + 157$

$140 + 45 = \underline{\quad}$

d.  $168 + 17$

$160 + 25 = \underline{\quad}$

# The Bridge to Ten Strategy Extend

Name: \_\_\_\_\_

**WORK OUT** **5**

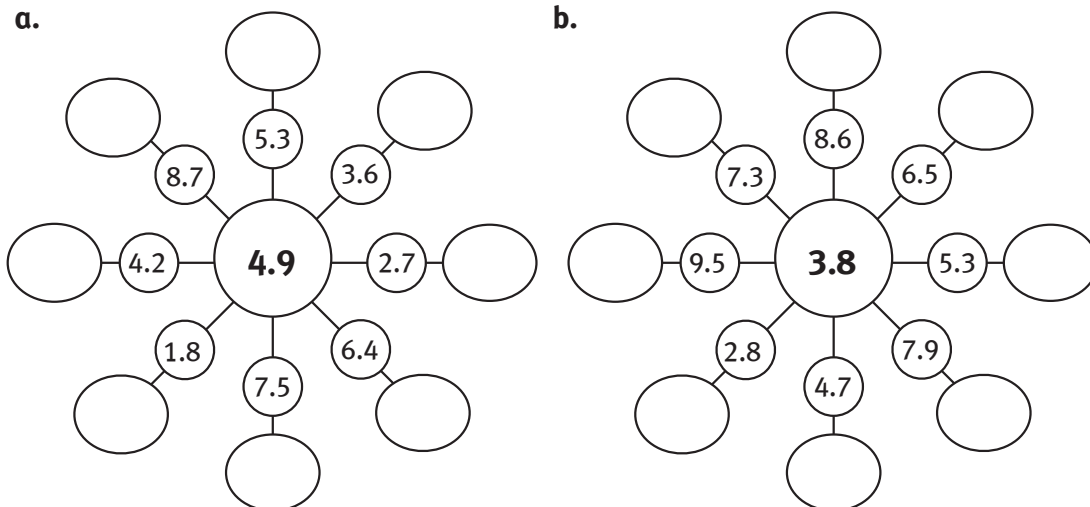
1. For each of these, write an easier number sentence that will help you figure out the problem below. Write the answer.

a.	$\underline{\quad} + \underline{\quad} = \underline{\quad}$ <b>SO</b> $4.6 + 3.8 = \underline{\quad}$	b.	$\underline{\quad} + \underline{\quad} = \underline{\quad}$ <b>SO</b> $8.9 + 4.3 = \underline{\quad}$	c.	$\underline{\quad} + \underline{\quad} = \underline{\quad}$ <b>SO</b> $7.8 + 5.4 = \underline{\quad}$
d.	$\underline{\quad} + \underline{\quad} = \underline{\quad}$ <b>SO</b> $1.9 + 6.5 = \underline{\quad}$	e.	$\underline{\quad} + \underline{\quad} = \underline{\quad}$ <b>SO</b> $2.8 + 3.6 = \underline{\quad}$	f.	$\underline{\quad} + \underline{\quad} = \underline{\quad}$ <b>SO</b> $9.9 + 6.7 = \underline{\quad}$

2. Write the answers. Place a ✓ above the numbers you adjusted.

a. $6.9 + 8.4 = \underline{\quad}$	b. $5.3 + 7.8 = \underline{\quad}$	c. $8.9 + 4.4 = \underline{\quad}$
d. $7.7 + 8.8 = \underline{\quad}$	e. $6.5 + 3.9 = \underline{\quad}$	f. $4.8 + 4.9 = \underline{\quad}$

3. Add the numbers on the spokes to the number in the center. Write the answers around the outside.



# The Bridge to Ten Strategy Extend

Name: \_\_\_\_\_

**WORK OUT** **5**

1. Adjust each of these to make a new sentence that is easier to figure out. Write the answer.

a.  $23.9 + 15.6$   
is the same as  
\_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_

b.  $42.7 + 14.8$   
is the same as  
\_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_

c.  $31.8 + 16.9$   
is the same as  
\_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_

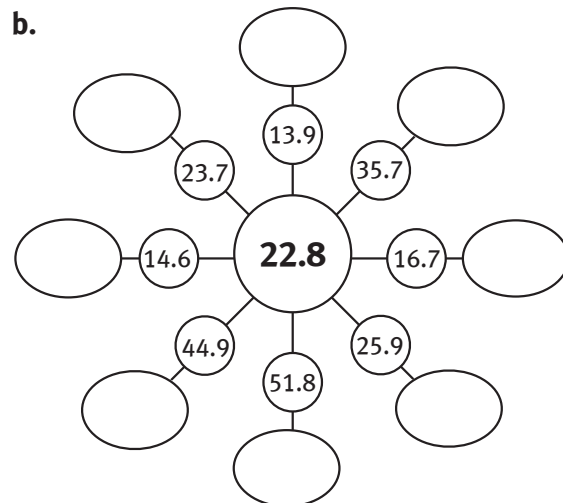
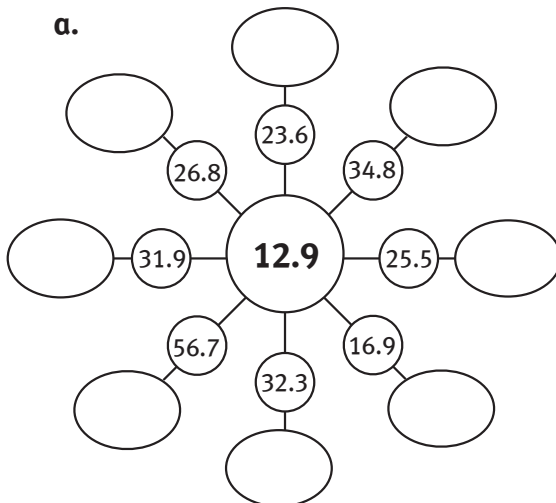
d.  $53.8 + 24.7$   
is the same as  
\_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_

2. Write the answers. Place a ✓ above the numbers you adjusted.

a.  $24.3 + 13.9 =$  \_\_\_\_\_    b.  $16.8 + 31.7 =$  \_\_\_\_\_    c.  $45.9 + 11.6 =$  \_\_\_\_\_

d.  $32.7 + 15.8 =$  \_\_\_\_\_    e.  $53.9 + 34.8 =$  \_\_\_\_\_    f.  $26.7 + 42.9 =$  \_\_\_\_\_

3. Add the numbers on the spokes to the number in the center.  
Write the answers around the outside.



# The Bridge to Ten Strategy

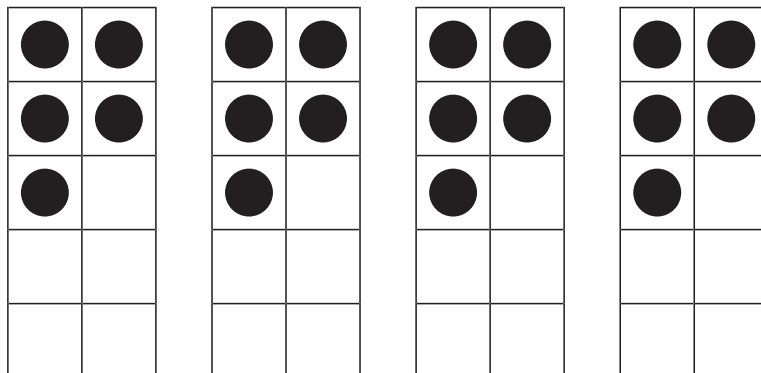
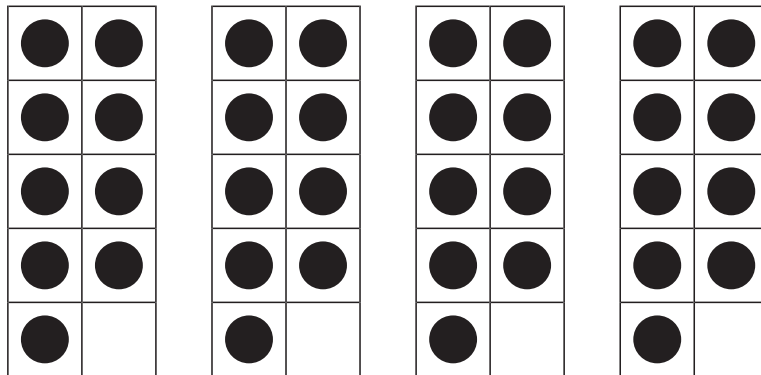
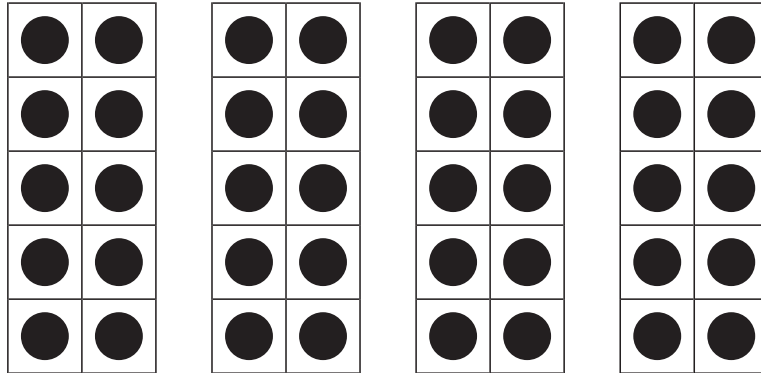
## First to Forty

The image displays four identical ten-frame grids arranged vertically. Each grid is a 2x5 array of empty squares. The grids are separated by horizontal lines. There are gray triangular shapes pointing towards the grids: one on the left side of the top grid, one on the right side of the top grid, one on the left side of the second grid, one on the right side of the second grid, one on the left side of the third grid, one on the right side of the third grid, one on the left side of the fourth grid, and one on the right side of the fourth grid.

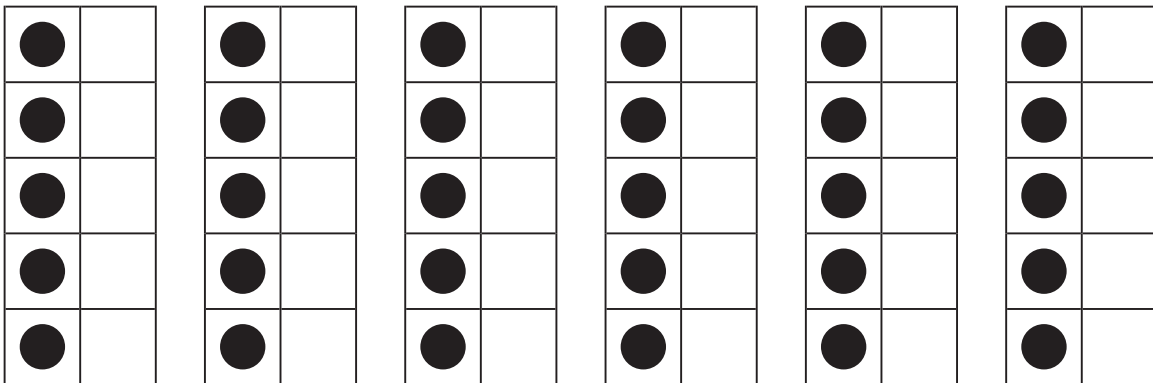
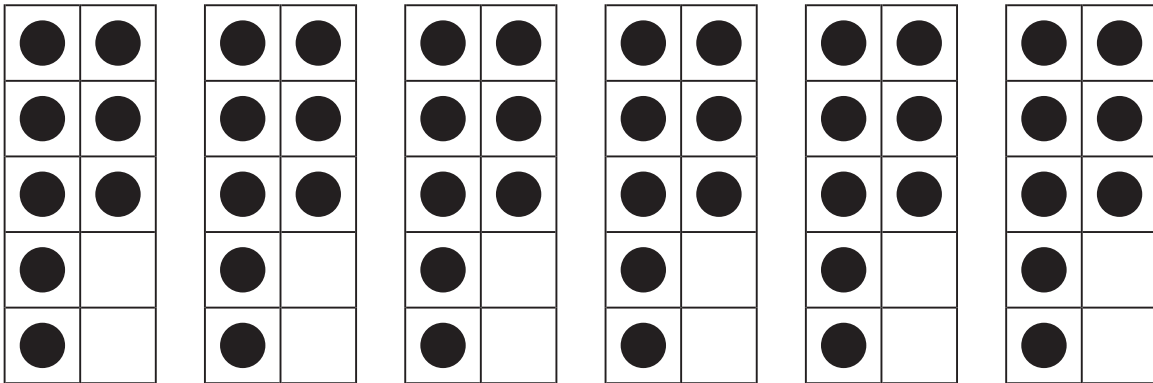
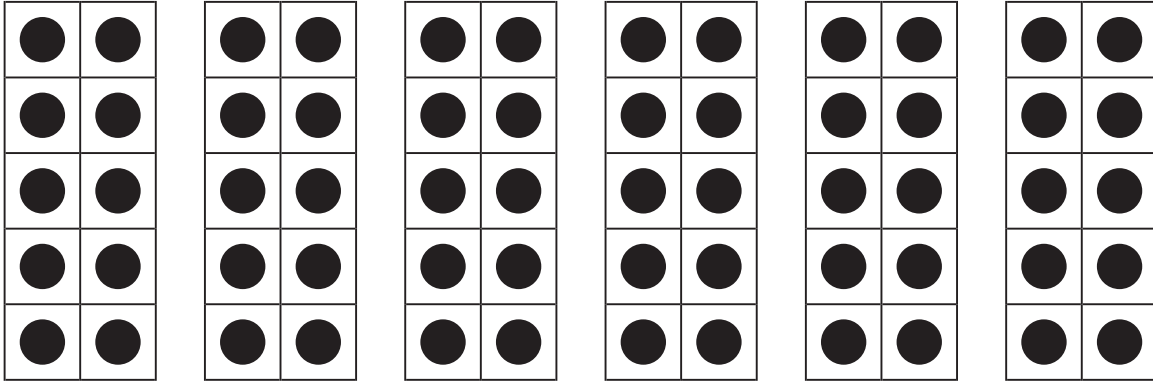
# Multiplication

**Materials:** Copy the ten frames pictures below and on the next page onto overhead transparencies.

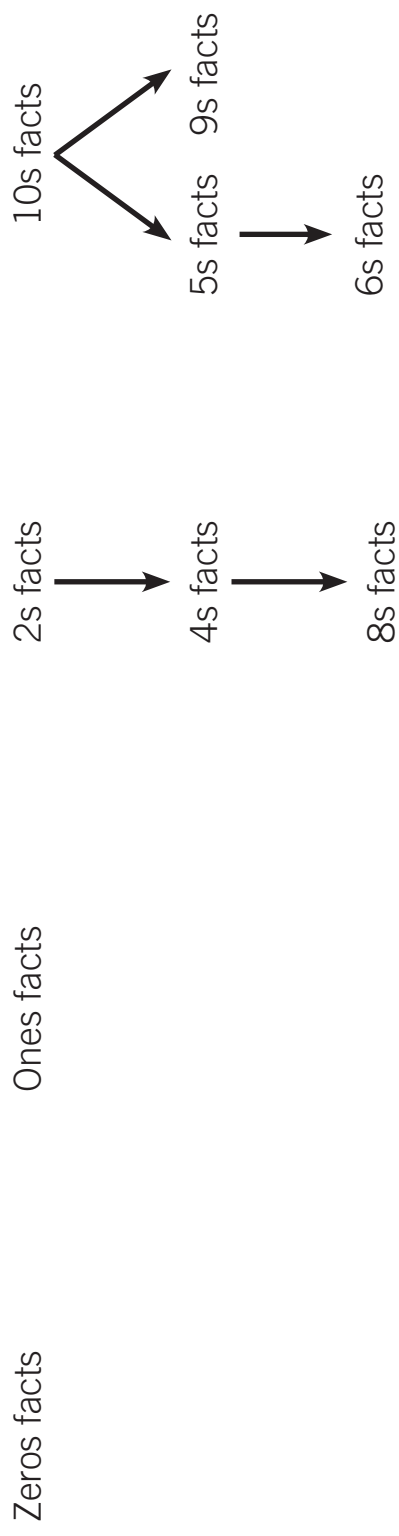
**Instructions:** Show each row of ten frames on the projector, one at a time by turning the projector on and off quickly. If needed show the picture again by quickly turning the projector on and off. Ask the students to tell the total and how they know.

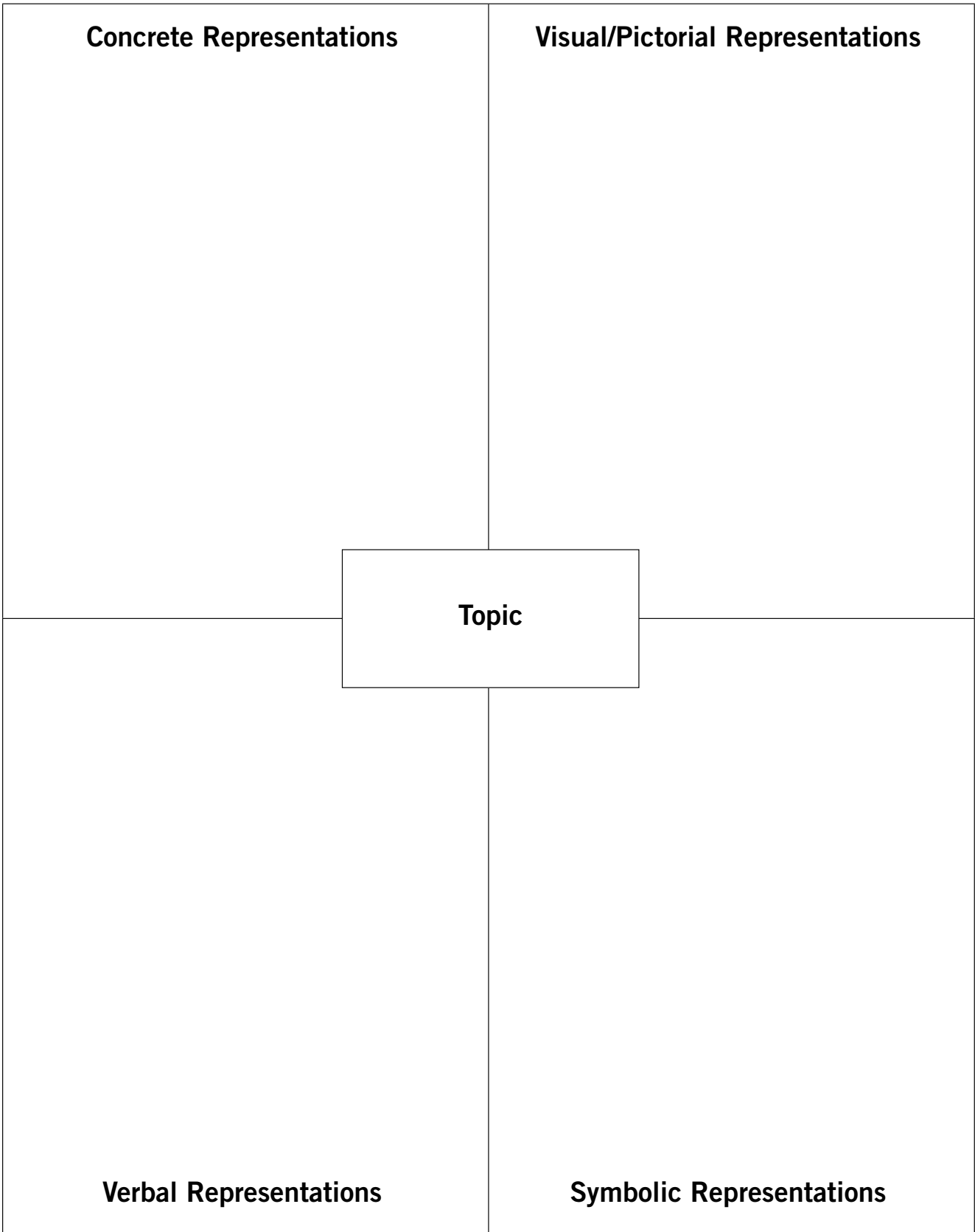


# Multiplication

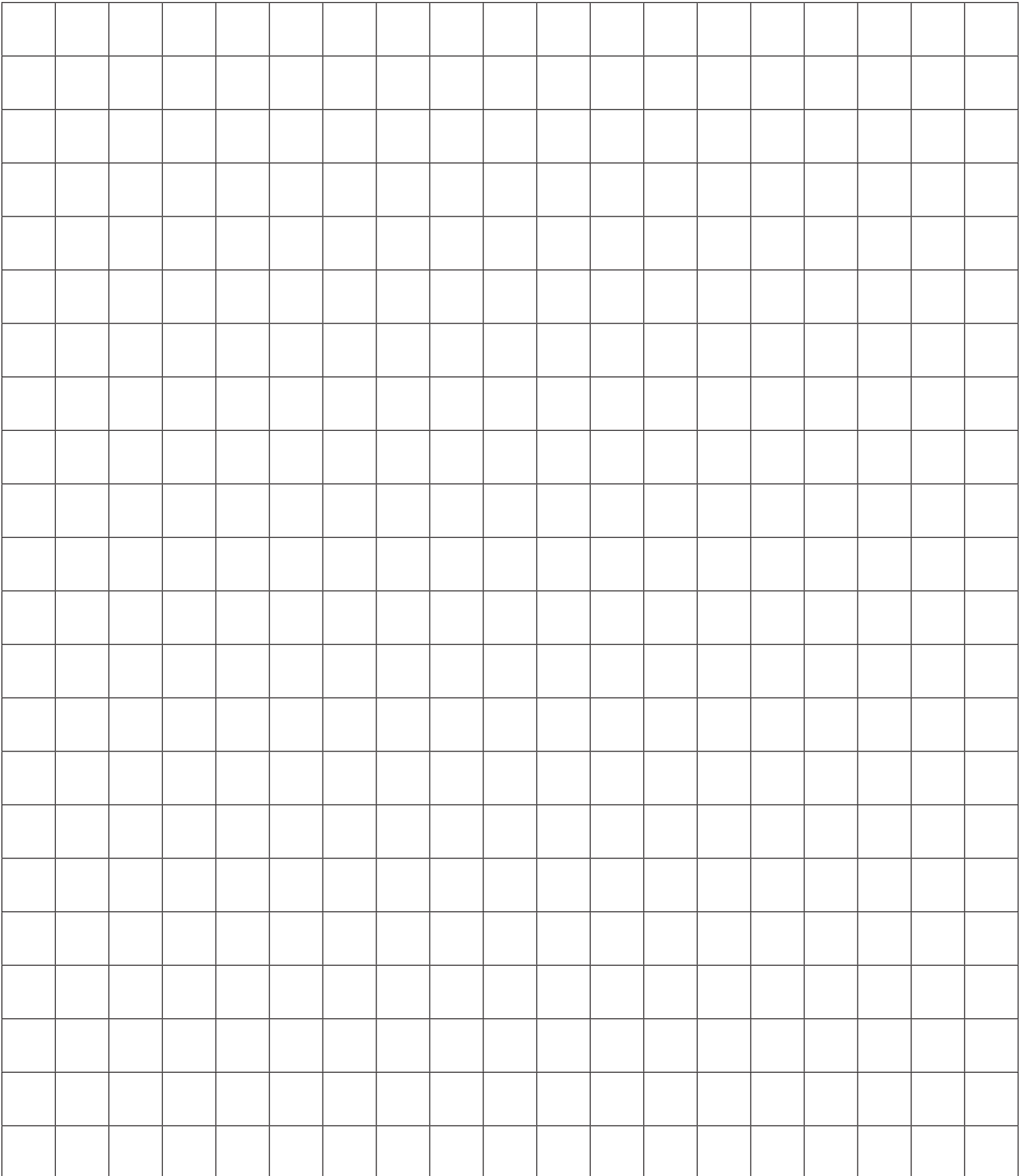


# Sequencing multiplication fact experiences





# ***Key models for building connections the area model***



# ***Summarizing the process***

## Planning for Intervention

- Identify common misconceptions
- Identify needed conceptual understandings
- Determine how far back to plug the hole
- Identify key visual models
- Link concrete, visual/pictorial, verbal, and symbolic representations
- Plan to bridge to grade level using similar models
- Be purposeful in material use and model selection
- Spiral

## Classroom Protocols

- Allow for mistakes—Don't panic!
- Ask versus tell (whenever possible)
- Use Think-pair-share and other informal interview techniques to determine understanding
- Ask questions to determine understandings versus reaching an instructional goal
- Require students to always explain thinking
- Build connections