



Developing Algebra Sense in the Elementary Years

PROFESSIONAL DEVELOPMENT

www.origomath.com

James Burnett, M.Ed.

ORIGO[®]
EDUCATION

pd

Developing Algebra Sense in the Elementary Years

PROFESSIONAL DEVELOPMENT

James Burnett

As President of ORIGO Education, James strives to lift the profile of mathematics through dynamic professional development and the development of quality, research-based classroom materials. James frequently presents workshops and speaks at conferences throughout North America, Australia, and New Zealand and has authored more than 150 mathematics books for teachers, and students aged 6–12.

Introduction

For many adults and indeed elementary school teachers, just the thought of algebra often conjures up feelings of anxiety and stress. It is often seen as being highly symbolic with little or no relevance to everyday life. This paper aims to show how algebra can be developed as a way of thinking that supports mathematics standards, and in so doing, demystify algebra for all.

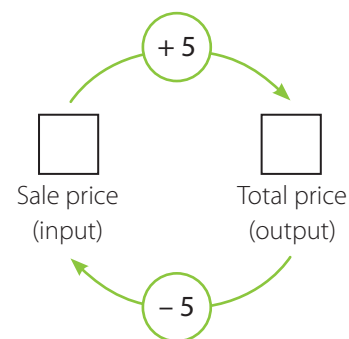
Algebra is more than a decontextualized manipulation of symbols. As Moses (1997) explained, “it (algebra) is a way of thinking, a method of seeing and expressing relationships. It is a way of generalizing” (p. 264). This “way of thinking” is evidenced in our day-to-day lives. For example, suppose you want to purchase a novel from an on-line bookstore on the Internet. The price of the book will vary as it depends on the title you choose. For this reason, you could call this amount a “variable”. The on-line store will no doubt charge an amount for freight. Often it is just a small flat rate cost, such as \$5. In this case, the freight charge remains the same, no matter what the cost of the book. You could call this amount a “constant”. There is now a relationship between the total amount paid and the cost of the book. Once identified, this relationship could be expressed in words or as symbols or a combination of both, for example:

$$\text{Total} = \text{Cost of the item} + \$5$$

The following month, the credit card statement may show a charge at the on-line bookstore for \$34. What was the cost of the book that was purchased? Fortunately, the generalization or rule that was identified can now be backtracked as follows:

$$\text{Cost of the item} = \text{Total} - \$5$$

As the diagram below shows, this relationship can also be expressed as a simple function, or function “machine”, that when given an input number (cost of item), can apply a rule and determine the output number (total). Alternatively, if given the output number, it can backtrack the rule to determine the input number.



Contemporary curricula recognize the ability to identify relationships, make generalizations and to backtrack those generalizations as major skills in algebra—even for the young elementary school student.

The Big Ideas

There are essentially two big ideas in algebra. The description of these ideas varies, but the goal remains the same. That is, we want students to develop deep understandings of

- Equivalence and equations, and
- Patterns and functions

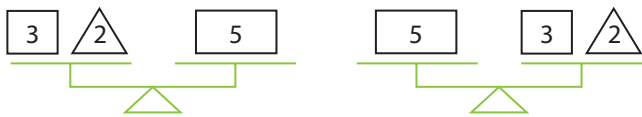
So where to start? One way is to think carefully about the types of problems we present students. Teachers have always considered the problem types associated with the various models of addition, subtraction (take-away, missing addend, comparison/difference), multiplication (set/group, array, linear/number line, cross product/combinations), and division (partition, quotient). But mathematical operations can also be classified as being either *static* or *active*. In short, static problems are used to develop the ideas of equivalence and equations. Active problems support the development of patterns and functions.

Static Problems

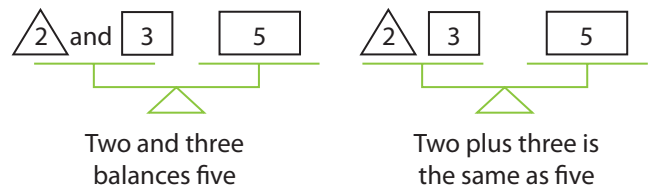
Static problems do not suggest any action. The action is often implied in the context of the situation. This is an example involving addition:

Brooke has \$2 in her hand and \$3 in her purse. How much money does she have altogether?

Problems such as this are often called *balance* problems as they can be represented as parts on a pan balance (as shown below). Using this model, the answer can also be displayed on the left—demonstrating the symmetric property of equations.



As illustrated in the following example, a carefully planned teaching sequence should move towards the formal written equation. The students will use their own natural language to describe what they see.



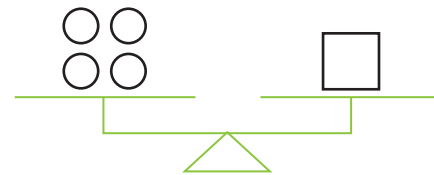
$$\triangle 2 + \square 3 = \square 5$$

Two plus three equals five

Multiplication can also be static. Consider the following problem.

There are 4 baskets with 3 eggs in each. How many eggs in all?

Again the action is only implied. This is one way to represent the problem on a pan balance.

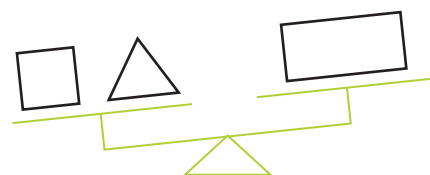


If each circle is worth three, what is the value of the square?

To move towards an equation, the next logical step is to use symbols to simplify or abbreviate the picture as illustrated below.



It is important to note that not all problems involve equality. The big idea of equivalence and equations also includes inequality. As shown below, the pan balance still provides a great visual aid for situations involving inequality.



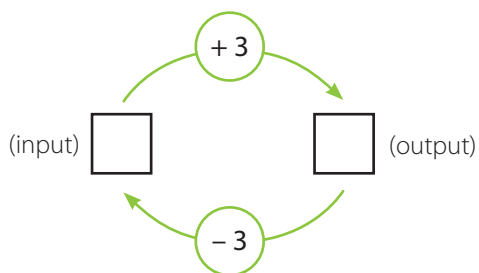
What generalization can you write?

Active Problems

As the language suggests, active problems indicate some kind of action. For addition, an active problem involves joining one part to another. For example,

Brooke has a bag with 6 apples. She puts 3 more apples in the bag. How many apples are in the bag now?

In active problems, the amount represented by a part is changed by adding one or more other amounts to it. For this reason, they are sometimes called *change* problems. Active or change problems provide an opportunity to introduce the big idea of function. In the above problem, the function is to add 3. It can be represented in much the same way as the problem involving the on-line bookstore in the introduction of this paper. The simple illustration below shows how the function can be reversed or backtracked. An input/output table will serve to make the problem more open while reinforcing the vital connection to the inverse operation—subtraction.



Rule: + _____

Input	Output
7	
27	
149	
	7
	27

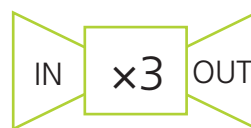
A simple sequence of activities may be as follows:

- Calculate the output number, given the function or rule and the input number.
- Calculate the input number, given the rule and the output number.
- Calculate the rule given a series of input and output numbers.

Multiplication is the operation that represents the greatest challenge to make active. When writing such a problem, first check that it can be reversed or backtracked. For example,

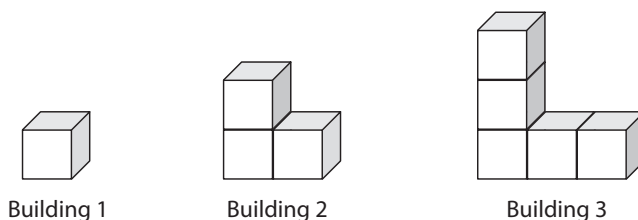
Brooke is packing tubes of tennis balls into a box. There are 3 balls in each tube. If she packs 12 tubes, how many tennis balls are in the box?

This problem can be represented using the diagram below. The rule is “x 3”, so if 12 go “in” the output number is 36. Suppose the output number is 18. The rule can now be reversed to determine that 6 tubes were placed in the box. This situation involves quotitive division, as the number in each group is known and the number of groups is unknown.



Patterns

Visit any class of 5 or 6 year olds and you will often see engaging patterning activities that range from simply finding and describing patterns, copying a pattern, and creating or extending a pattern, to high-level activities such as finding a missing element within a pattern or translating a given pattern using a different medium. Activities such as these are important, but to move from patterns to algebra, good quality questions are needed to bring out the generalizations and relationships that involve algebraic thinking. Three types of patterns will be investigated here. Consider first the following *growing* pattern.




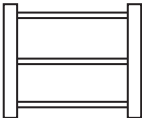
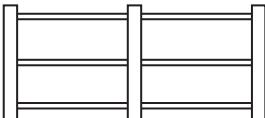
The questions that are asked should encourage the students to focus on the position in the pattern or sequence of buildings and to generate a rule that relates to the position. For example,

What does the 8th building look like? How many blocks are in that building?

Middle school students should be able to study the pattern and generalize that the total number of blocks in any building is double the number of blocks along the base, less one. This rule can now be backtracked to answer questions such as,

Which building has a total of 13 blocks?

The same applies to a *relationship* pattern. Look at the pattern below and complete the table.

	Number of posts	Number of rails
	1	0
	2	
	3	
	4	

Warren (1996) indicated that most adults and students look down the second column to complete the number of rails in the fourth picture. This strategy works fine when you just want to complete the data for the next picture. However, a good question will have the students look for a relationship between the number of posts and the number of rails in any one picture. For example,

How many rails will be required for 12 (... 15 or ... 20) posts? What rule can be used to work out the number of rails required for any number of posts?

The third and arguably the most common type of pattern that is explored in classrooms is the *repeating* pattern.

Identify the repeating element in the pattern below, then consider the following questions. What is the 50th shape in the pattern? How do you know? What mathematics did you use?



Students will need to generalize when the focus of the question is on the position of an element within the pattern.

Conclusion

This paper has provided practical suggestions for investigating the big ideas in algebra. In short, four important points were made.

- Algebra is a “way of thinking” that supports other mathematics standards.
- Static problems can be used to develop understandings in equivalence and equations. Active problems can be used to address patterns and functions.
- Good quality questions are needed to help students move from patterns to algebra.
- Algebra need not be scary—it can be engaging and fun!

References

- Moses, B. (1997). Algebra for a new century. *Teaching Children Mathematics*, 3(6), 264–5.
- Burnett, J. & Irons, C. (2005). *QLD GO Maths*. ORIGO Education: Brisbane.
- Warren, E. (2006). *Algebra for all*. ORIGO Education: Brisbane.
- Warren, E. (1996). *Interactions between instructional approaches, students’ reasoning processes and their understanding of elementary algebra*. PhD thesis. Queensland University of Technology: Brisbane.