

DESIGNING EFFECTIVE MATHEMATICS INTERVENTION EXPERIENCES

PROFESSIONAL DEVELOPMENT MODULE

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Designing Effective Mathematics Intervention Experiences
Professional Development Module

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What mathematics do your students...

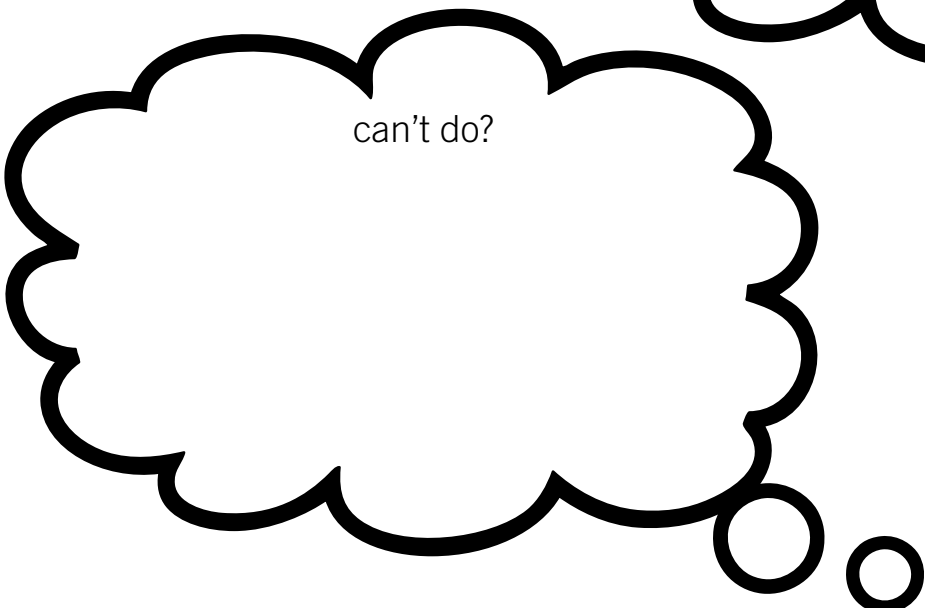
struggle with?



don't understand?



can't do?



In column 1, record the misconceptions related to the previously identified struggles.

Misconceptions (What is it the students don't understand?)	Needed Conceptual Understandings (If a student really understood _____ they would understand...)

For each misconception listed in column 1, record the related conceptual understanding needed for students to be successful in mathematics.

What do you think?

A student was solving some simple math fact problems in her mathematics journal. This is what she wrote.

1. $4 + 5 = 5$

2. $4 + 6 = 6$

3. $4 + 7 = 7$

4. $4 + 8 = 8$

5. $4 + 9 = 9$

What is the student's error?

What is the disconnect?

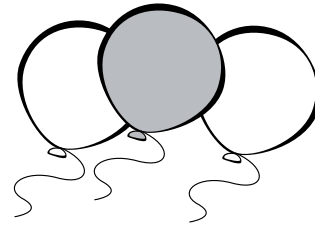
How would you intervene?

The Bridge to Ten Strategy Reinforce

WARM UP **11**

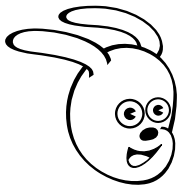
Name: _____

Jade had 9 girls and 6 boys at her party.
How many guests in all?

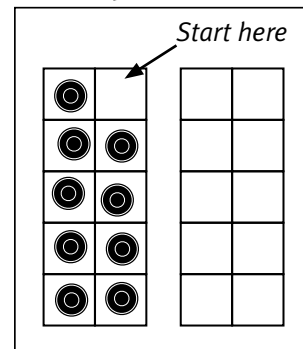


1. a. These ten-frames show 9 counters.
Draw 6 more counters.

- b. Write the total. _____



What did you notice?

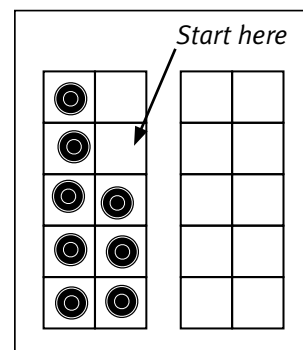


- c. Complete the sentence.

$9 + 6$ is the same as $10 +$ _____

2. a. These ten-frames show 8.
Draw 4 more counters.

- b. Write the total. _____



- c. Complete the sentence.

$8 + 4$ is the same as $10 +$ _____

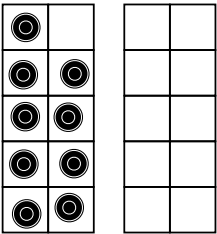
The Bridge to Ten Strategy Reinforce

WORK OUT 11

Name: _____

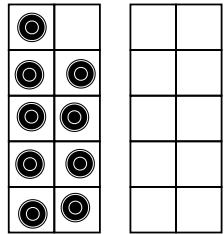
1. Draw more counters then complete the sentence.

a. Draw 7 more



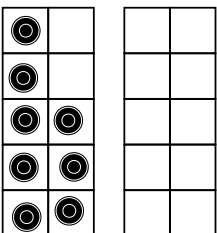
$\overset{\circ}{9} + \underline{\quad}$
is the same as
 $\overset{\circ}{10} + \underline{\quad} = \underline{\quad}$

b. Draw 5 more



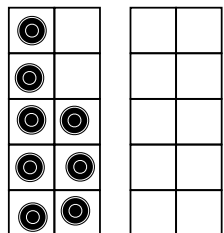
$\overset{\circ}{9} + \underline{\quad}$
is the same as
 $\overset{\circ}{10} + \underline{\quad} = \underline{\quad}$

c. Draw 5 more



$\underline{\quad} + \underline{\quad}$
is the same as
 $\underline{\quad} + \underline{\quad} = \underline{\quad}$

d. Draw 3 more



$\underline{\quad} + \underline{\quad}$
is the same as
 $\underline{\quad} + \underline{\quad} = \underline{\quad}$

2. For each of these, draw an arrow to a number sentence below that has the same answer. Write the answer.

a. $9 + 8$

b. $8 + 6$

c. $9 + 3$

$10 + 2 = \underline{\quad}$

$10 + 7 = \underline{\quad}$

$10 + 4 = \underline{\quad}$

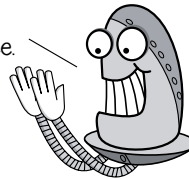
The Bridge to Ten Strategy Practice

DO YOU KNOW?

What has bread on both sides and is afraid of everything?

Figure out each of these and draw a straight line to the matching answer. The line will pass through a number and a letter. Write the letter above its matching number at the bottom of the page.

The answers can be used more than once.



$3 + 9$		ANSWERS		$3 + 8$
$5 + 8$	(E)	11	(D)	$9 + 3$
$9 + 5$	(31)	12	(21)	$8 + 5$
$8 + 3$	(H) (I)	13	(36)	$5 + 9$
$8 + 4$	(33)	14	(I) (N)	$4 + 8$
$6 + 8$	(K) (26)	15	(C)	$6 + 9$
$9 + 6$	(C) (A)		(22)	$9 + 4$
$4 + 9$			(28)	$8 + 6$

E

 21 22 23 24 25 26 27 28

D

 29 30 31 32 33 34 35 36

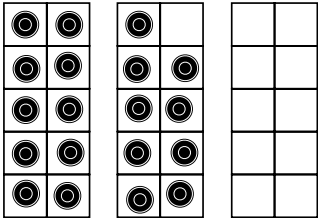
The Bridge to Ten Strategy Extend

WORK OUT 12

Name: _____

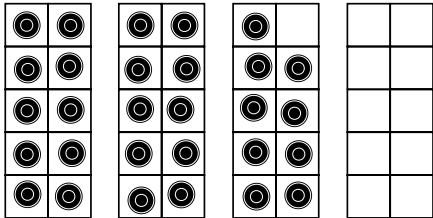
1. For each of these, draw more counters then complete the sentence.

a. Draw 7 more



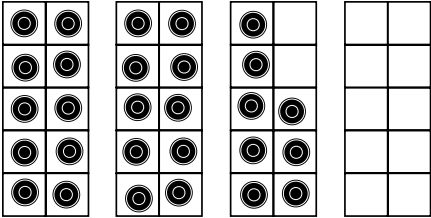
$19 + \underline{\quad}$
 is the same as
 $20 + \underline{\quad} = \underline{\quad}$

b. Draw 6 more



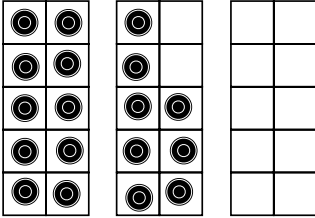
$29 + \underline{\quad}$
 is the same as
 $\underline{\quad} + \underline{\quad} = \underline{\quad}$

c. Draw 5 more



$\underline{\quad} + \underline{\quad}$
 is the same as
 $\underline{\quad} + \underline{\quad} = \underline{\quad}$

d. Draw 8 more



$\underline{\quad} + \underline{\quad}$
 is the same as
 $\underline{\quad} + \underline{\quad} = \underline{\quad}$

2. For each of these, draw an arrow to a number sentence below that has the same answer. Write the answer.

a. $29 + 8$ b. $58 + 3$ c. $28 + 7$ d. $59 + 5$

$30 + 5 = \underline{\quad}$

$30 + 7 = \underline{\quad}$

$60 + 4 = \underline{\quad}$

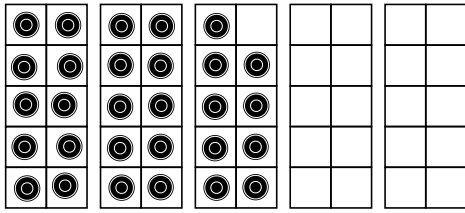
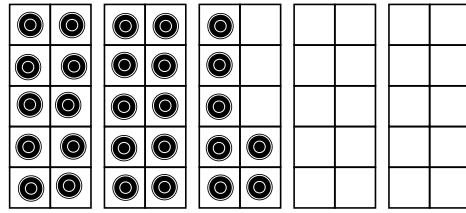
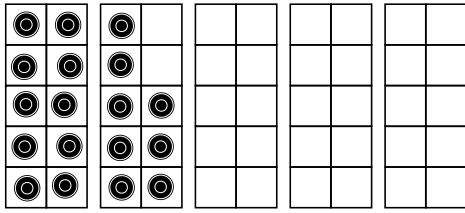
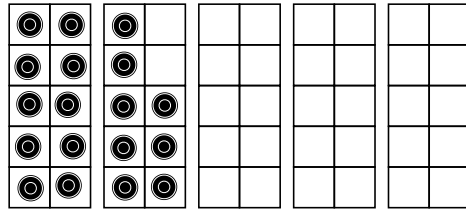
$60 + 1 = \underline{\quad}$

The Bridge to Ten Strategy Extend

Name: _____

WORK OUT **6**

1. For each of these, draw more counters then complete the sentence.

<p>a. Draw 16 more</p>  <p>29 + _____ is the same as 30 + _____ = _____</p>	<p>b. Draw 17 more</p>  <p>27 + _____ is the same as 30 + _____ = _____</p>
<p>c. Draw 27 more</p>  <p>_____ + _____ is the same as _____ + _____ = _____</p>	<p>d. Draw 17 more</p>  <p>_____ + _____ is the same as _____ + _____ = _____</p>

2. For each of these, draw an arrow to a number sentence below that has the same answer. Write the answer.

a. 47 + 18	b. 68 + 27	c. 69 + 33	d. 48 + 26
---	---	---	---

50 + 24 = _____	50 + 15 = _____	70 + 25 = _____	70 + 32 = _____
---	---	---	---

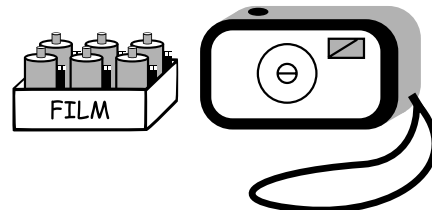
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The Bridge to Ten Strategy Extend

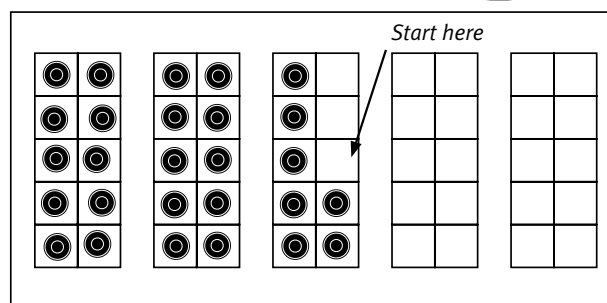
WARM UP **6**

Name: _____

Aden bought a camera for \$27
and a film pack for \$18.
How much did he spend in all?



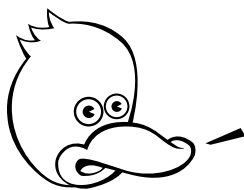
1. a. These ten-frames show 27 counters.
Draw 18 more counters.



- b. Write the total. _____

- c. Complete this sentence to match the picture above.

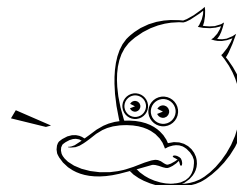
$27 + 18$ is the same as $30 +$ _____



What do you notice?

2. Use the same method to figure out $148 + 27$ in your head.

Try making a ten.



Complete this sentence to help you.

$148 + 27$ is the same as $150 +$ _____

The Bridge to Ten Strategy Extend

WORK OUT 6

Name: _____

1. For each of these, draw an arrow to a number sentence below that has the same answer. Write the answer.

a. $97 + 38$ b. $128 + 47$ c. $119 + 26$

$130 + 45 = \underline{\quad}$ $100 + 35 = \underline{\quad}$ $120 + 25 = \underline{\quad}$

2. Complete each sentence.

a. $68 + 27$
is the same as
 $70 + \underline{\quad}$

b. $247 + 36$
is the same as
 $250 + \underline{\quad}$

c. $169 + 26$
is the same as
 $170 + \underline{\quad}$

d. $38 + 127$
is the same as
 $\underline{\quad} + 130$

e. $26 + 139$
is the same as
 $\underline{\quad} + 140$

f. $47 + 329$
is the same as
 $\underline{\quad} + 330$

3. For each of these, draw an arrow to the number sentence you could use to figure it out. Write the answer.

a. $138 + 47$ $120 + 65 = \underline{\quad}$

b. $66 + 119$ $170 + 15 = \underline{\quad}$

c. $28 + 157$ $140 + 45 = \underline{\quad}$

d. $168 + 17$ $160 + 25 = \underline{\quad}$

The Bridge to Ten Strategy Extend

Name: _____

WORK OUT **5**

1. For each of these, write an easier number sentence that will help you figure out the problem below. Write the answer.

a.

_____ + _____ = _____
SO
4.6 + 3.8 = _____

b.

_____ + _____ = _____
SO
8.9 + 4.3 = _____

c.

_____ + _____ = _____
SO
7.8 + 5.4 = _____

d.

_____ + _____ = _____
SO
1.9 + 6.5 = _____

e.

_____ + _____ = _____
SO
2.8 + 3.6 = _____

f.

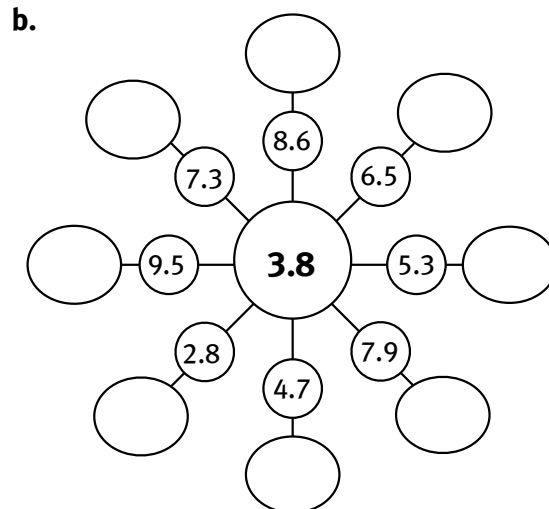
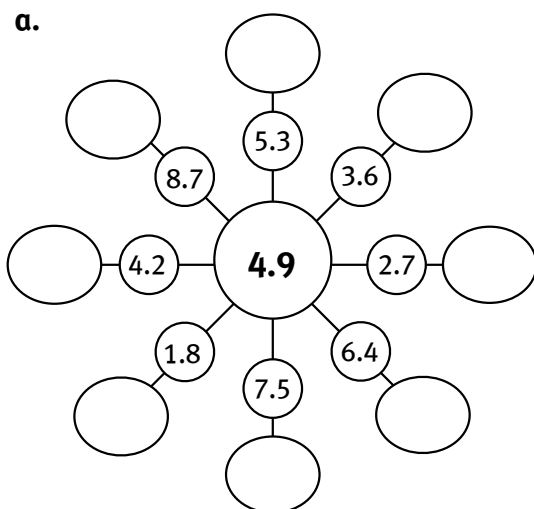
_____ + _____ = _____
SO
9.9 + 6.7 = _____

2. Write the answers. Place a ✓ above the numbers you adjusted.

a. $6.9 + 8.4 =$ _____ b. $5.3 + 7.8 =$ _____ c. $8.9 + 4.4 =$ _____

d. $7.7 + 8.8 =$ _____ e. $6.5 + 3.9 =$ _____ f. $4.8 + 4.9 =$ _____

3. Add the numbers on the spokes to the number in the center. Write the answers around the outside.



The Bridge to Ten Strategy Extend

Name: _____

WORK OUT **5**

1. Adjust each of these to make a new sentence that is easier to figure out. Write the answer.

a. $23.9 + 15.6$
is the same as
_____ + _____ = _____

b. $42.7 + 14.8$
is the same as
_____ + _____ = _____

c. $31.8 + 16.9$
is the same as
_____ + _____ = _____

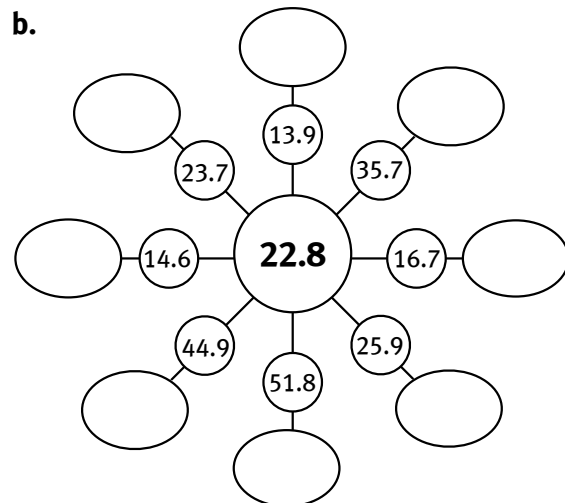
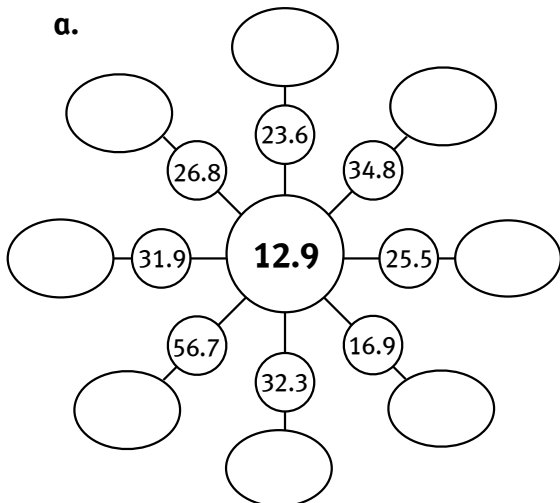
d. $53.8 + 24.7$
is the same as
_____ + _____ = _____

2. Write the answers. Place a ✓ above the numbers you adjusted.

a. $24.3 + 13.9 =$ _____ b. $16.8 + 31.7 =$ _____ c. $45.9 + 11.6 =$ _____

d. $32.7 + 15.8 =$ _____ e. $53.9 + 34.8 =$ _____ f. $26.7 + 42.9 =$ _____

3. Add the numbers on the spokes to the number in the center.
Write the answers around the outside.



The Bridge to Ten Strategy

First to Forty

The image displays four identical ten-frame grids arranged vertically. Each grid is a 2x5 array of empty squares. The grids are separated by horizontal lines. There are grey arrowheads pointing to the left and right of the grids, and a grey arrowhead pointing down between the first and second grids.

The 2s, 4s, and 8s Facts

Double Up

6	16	10	12
10	8	14	16
14	6	12	8

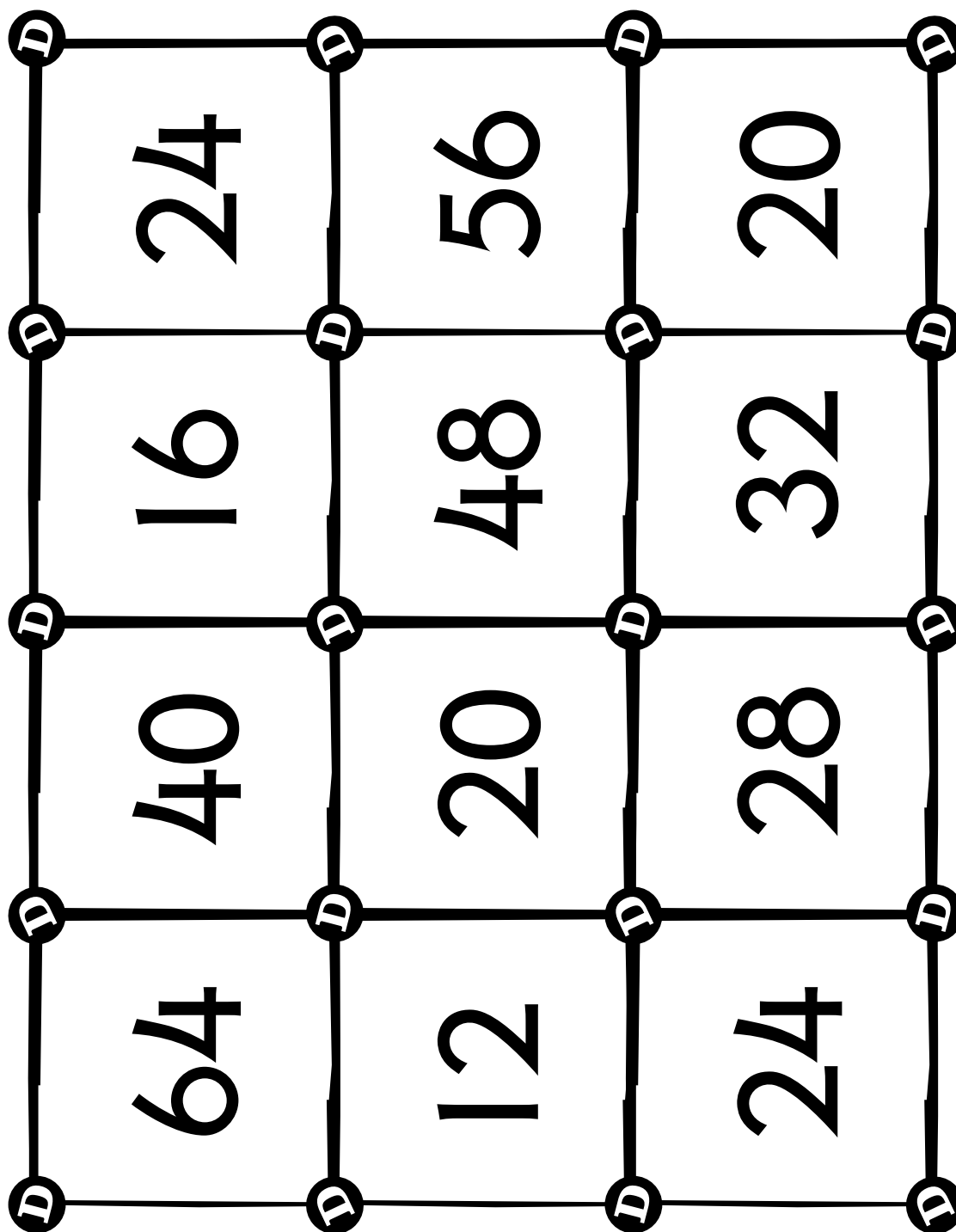
The 2s, 4s, and 8s Facts

Double Double

28	6	32	16
16	10	24	12
20	12	8	14

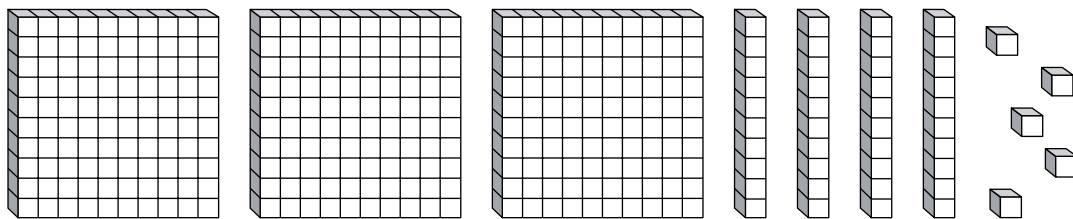
The 2s, 4s, and 8s Facts

Do the Ds

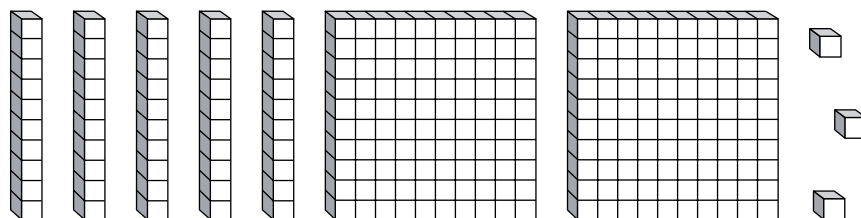


The importance of number sense

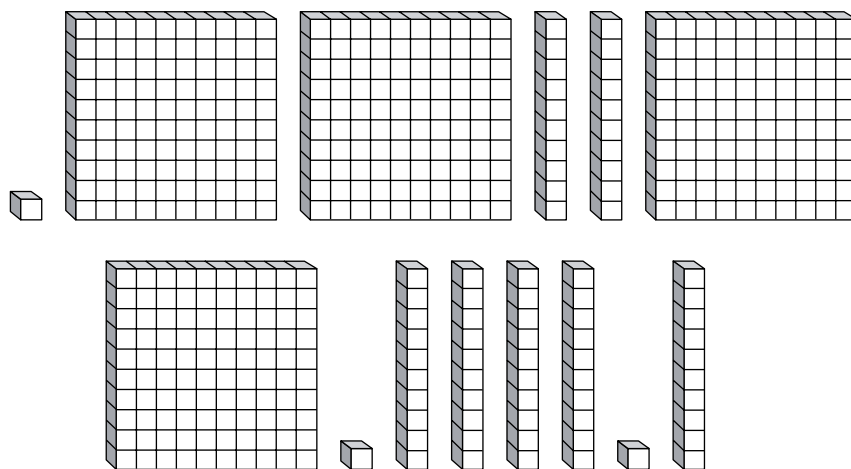
A look at equivalencies



If I only had  how many would I need?



What difficulties might your students have?



What activities could we do to build connections?

The importance of number sense

A look at equivalencies

Find all of the ways to make 3.42.

Tens	Ones	Tenths	Hundredths	Thousandths

Find all of the ways to make $2\frac{2}{3}$.

Ones	Halves	Thirds	Fourths	Sixths

Find all of the ways to make \$2.63.

Dollars	Quarters	Dimes	Nickels	Pennies

The importance of number sense

A look at equivalencies

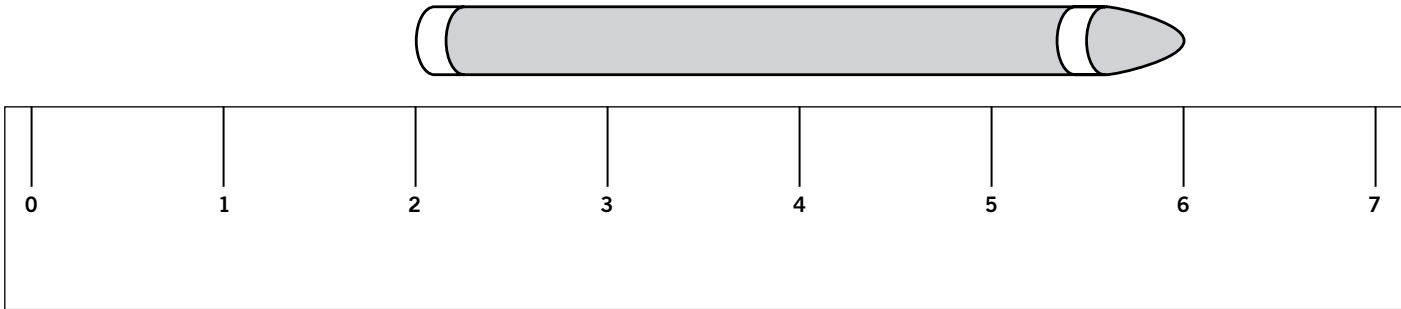
Find all of the ways to make 4365.

Thousands	Hundreds	Tens	Ones

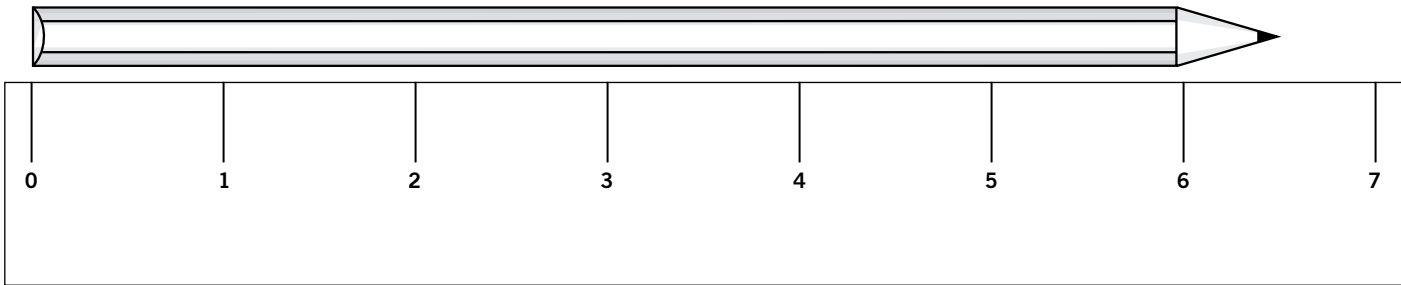
Find all of the ways to make 62.

Tens	Ones

Find all of the ways to make 10.

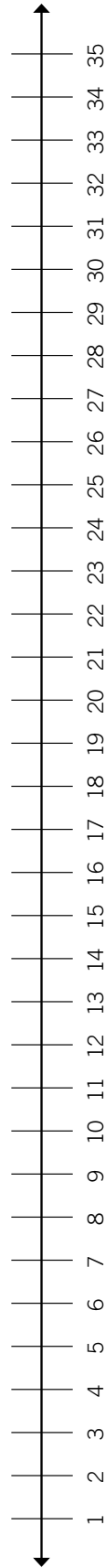
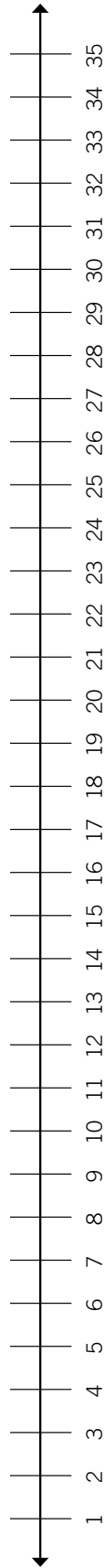
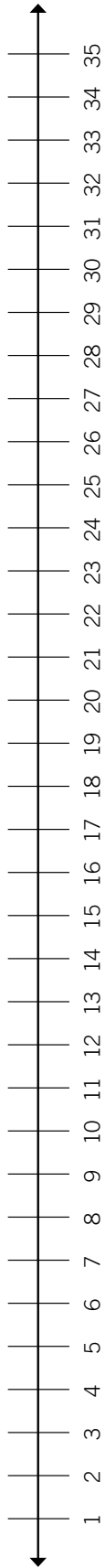
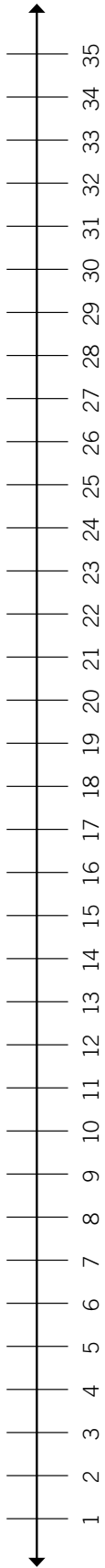


How long is the crayon?



How long is the pencil in half inches?

Key models for building connections the number line



Key models for building connections the number line

Up or Down

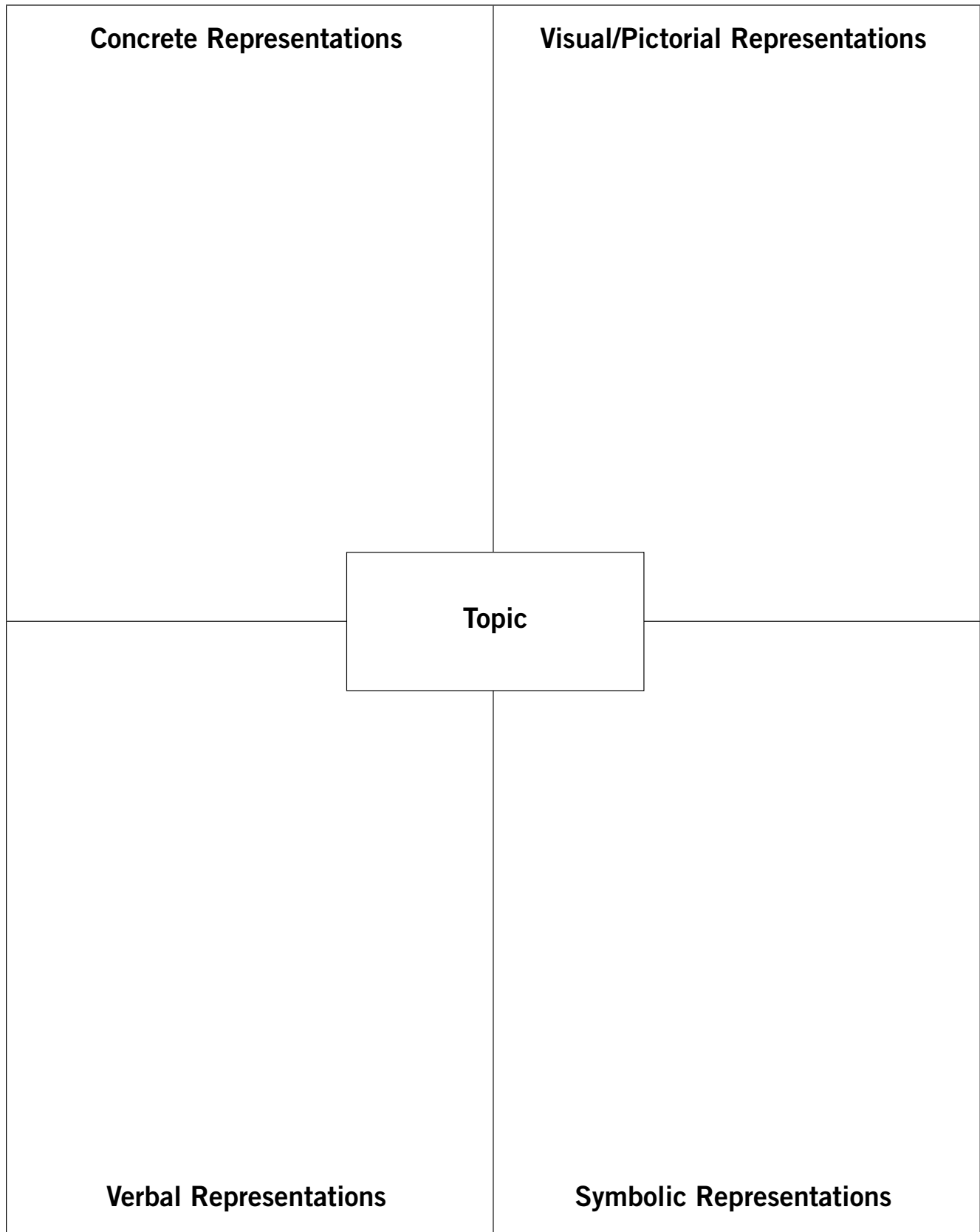
90 - 15	95 - 20	90 - 25	95 - 30	90 - 35
80 - 15	85 - 20	80 - 25	85 - 30	80 - 35
70 - 15	75 - 20	70 - 25	75 - 30	70 - 35
60 - 15	65 - 20	60 - 25	65 - 30	60 - 35
50 - 15	55 - 20	50 - 25	55 - 30	50 - 35
40 - 15	45 - 20	40 - 25	45 - 30	40 - 35

Key models for building connections the number line

Up or Down Again

75	75	65	65	55
65	65	55	55	45
55	55	45	45	35
45	45	35	35	25
35	35	25	25	15
25	25	15	15	5

Key models for building connections the number line



Key models for building connections the number line

1.
$$\begin{array}{r} 600 \\ - 432 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 401 \\ - 134 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 2003 \\ - 867 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 332 \\ - 198 \\ \hline \end{array}$$

5.
$$\begin{array}{r} 6.00 \\ - 4.32 \\ \hline \end{array}$$

6.
$$\begin{array}{r} 3.01 \\ - 1.57 \\ \hline \end{array}$$

Key models for building connections the number line

11

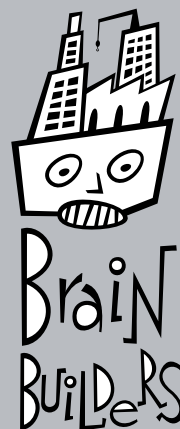
Write the answers that you can figure out **in your head**.

- a. $80 - 29 = \underline{\quad}$
- b. $400 - 299 = \underline{\quad}$
- c. $5,604 - 600 = \underline{\quad}$
- d. $165 - 75 = \underline{\quad}$
- e. $35 - 21 = \underline{\quad}$

Computation and Number Sense

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Green Tank



6

Look at this strategy.

$$13.4 - 5.9 \text{ is the same as } 13.5 - 6$$

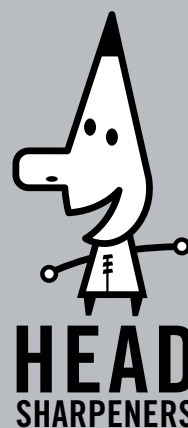
Copy and complete these. Try using the same strategy.

- a. $23.4 - 6.9 = \underline{\quad}$
- b. $15.3 - 2.8 = \underline{\quad}$
- c. $19.2 - 4.7 = \underline{\quad}$
- d. $16.6 - 7.8 = \underline{\quad}$

Computation and Number Sense

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Red Tank



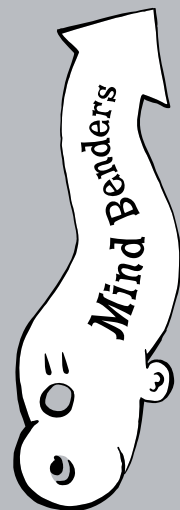
Key models for building connections the number line

14

- a. Figure out the answer to this number sentence
in your head.

$$3,200 - 1,900 = \underline{\hspace{2cm}}$$

- b. Write about the strategy you used.
c. Write **four** other number sentences that you could
solve the same way.



Computation and Number Sense

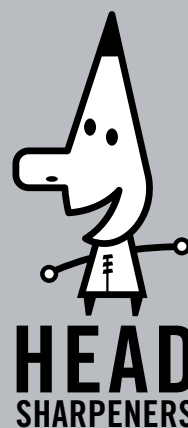
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Red Tank

12

Write the answers that you can figure out **in your head.**

- a. $\$20 - \$15.25 = \underline{\hspace{2cm}}$
b. $36 - 7.8 = \underline{\hspace{2cm}}$
c. $8.97 - 6.93 = \underline{\hspace{2cm}}$
d. $\frac{3}{4} - \frac{1}{2} = \underline{\hspace{2cm}}$
e. $350 - 20 - 0.8 = \underline{\hspace{2cm}}$

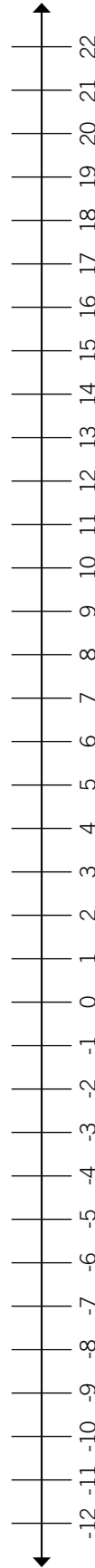
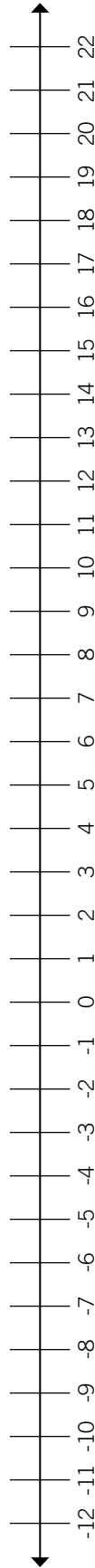
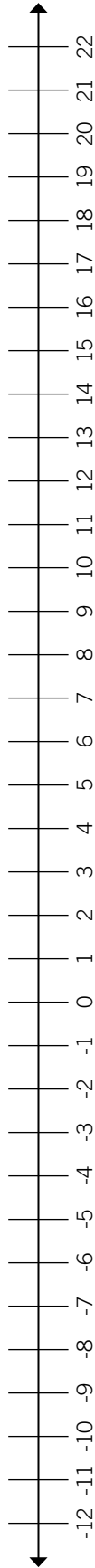
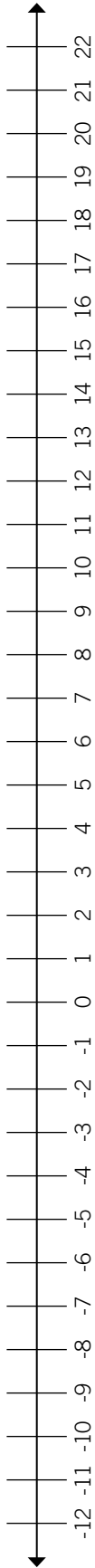


Computation and Number Sense

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Blue Tank

Key models for building connections the number line



Key models for building connections the number line

1. $4 - 1 =$

2. $1 - 4 =$

3. $2 - -5 =$

4. $-5 - 2 =$

5. $25 - -15 =$

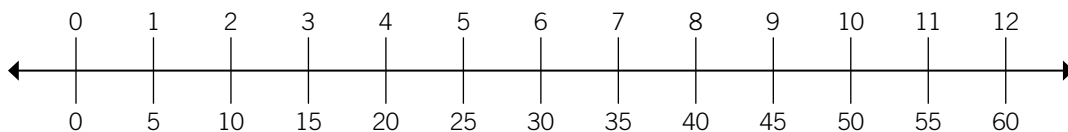
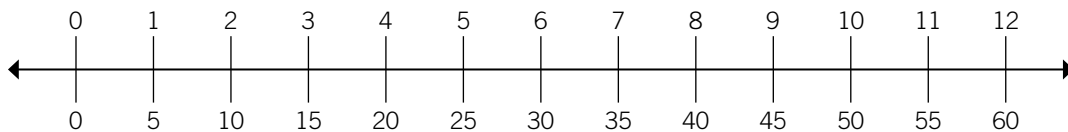
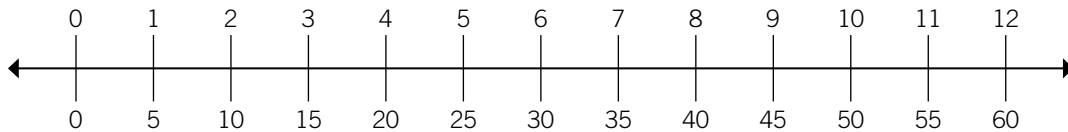
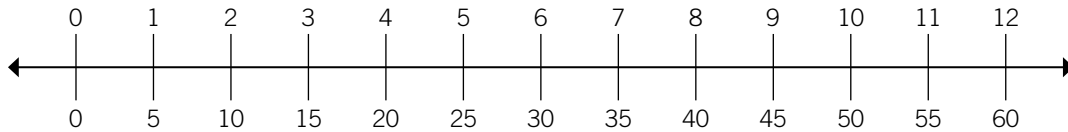
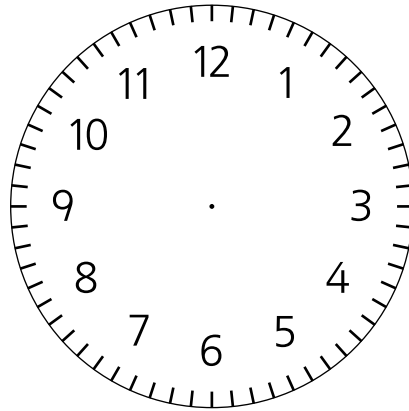
6. $-35 - 20$

Extensions

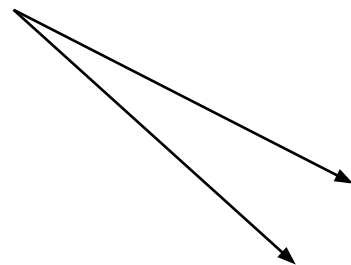
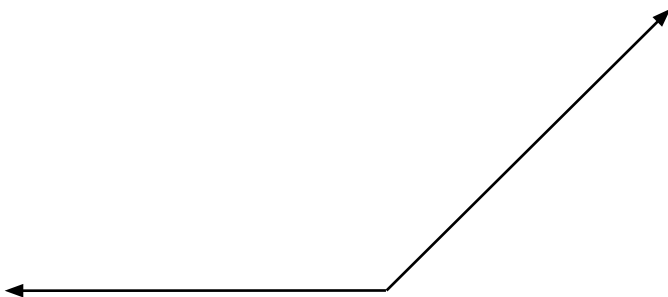
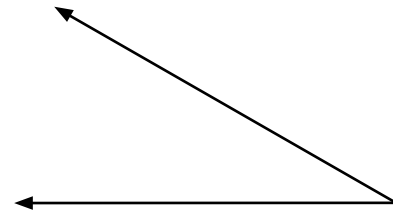
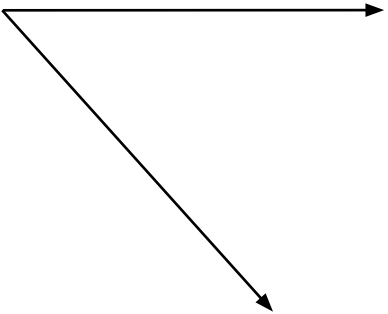
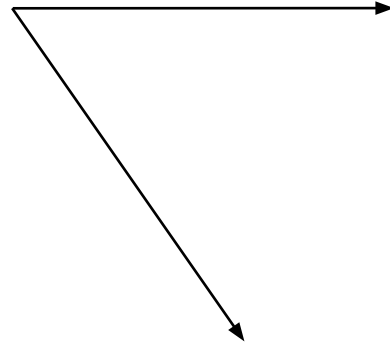
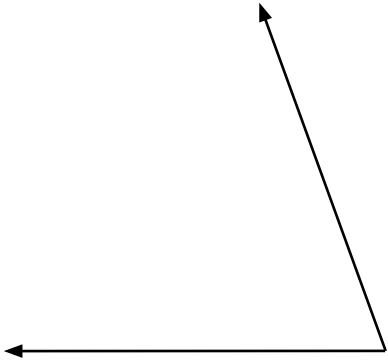
7. $|-3| =$

8. $|x| = 5$

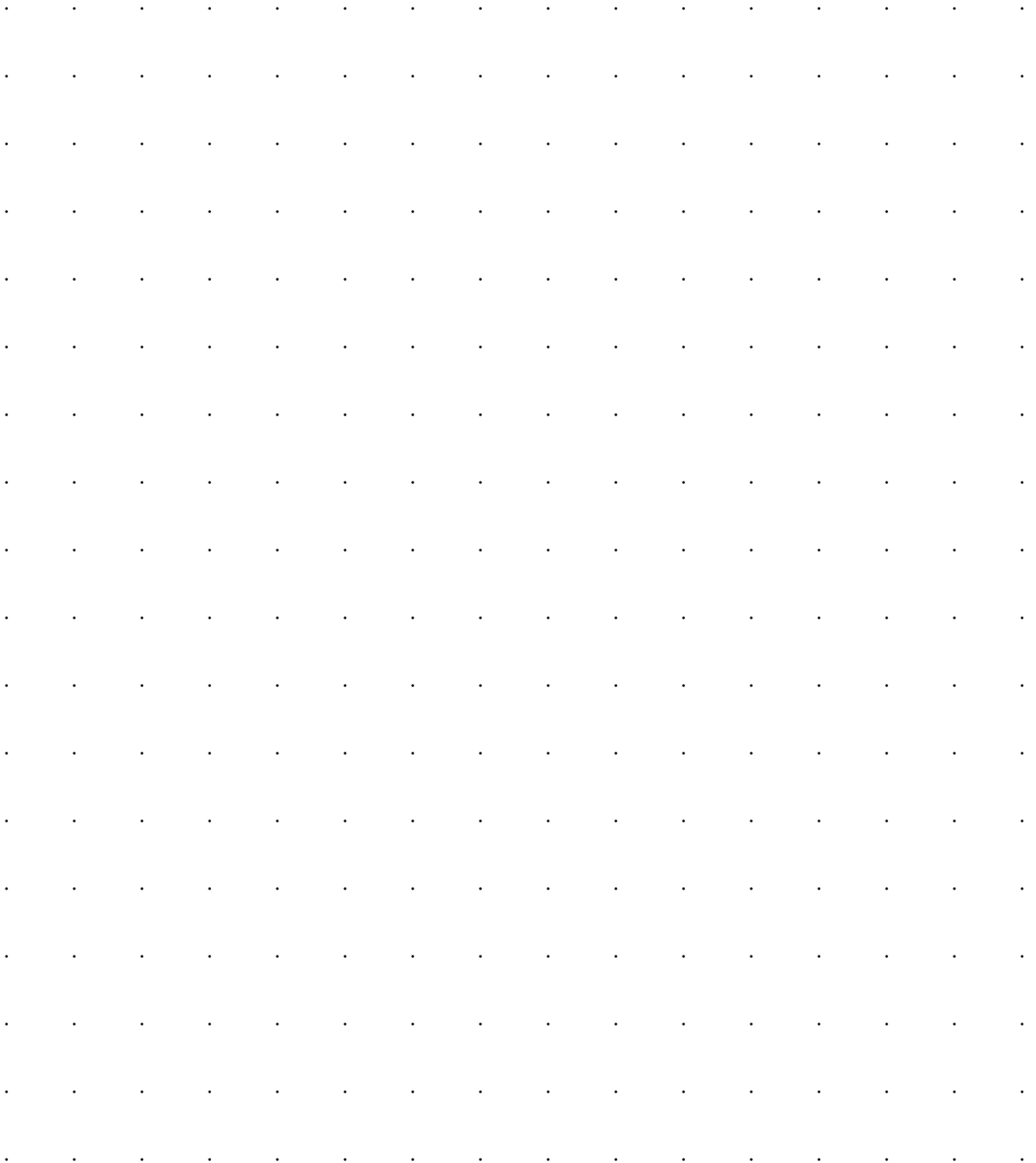
Key models for building connections the number line



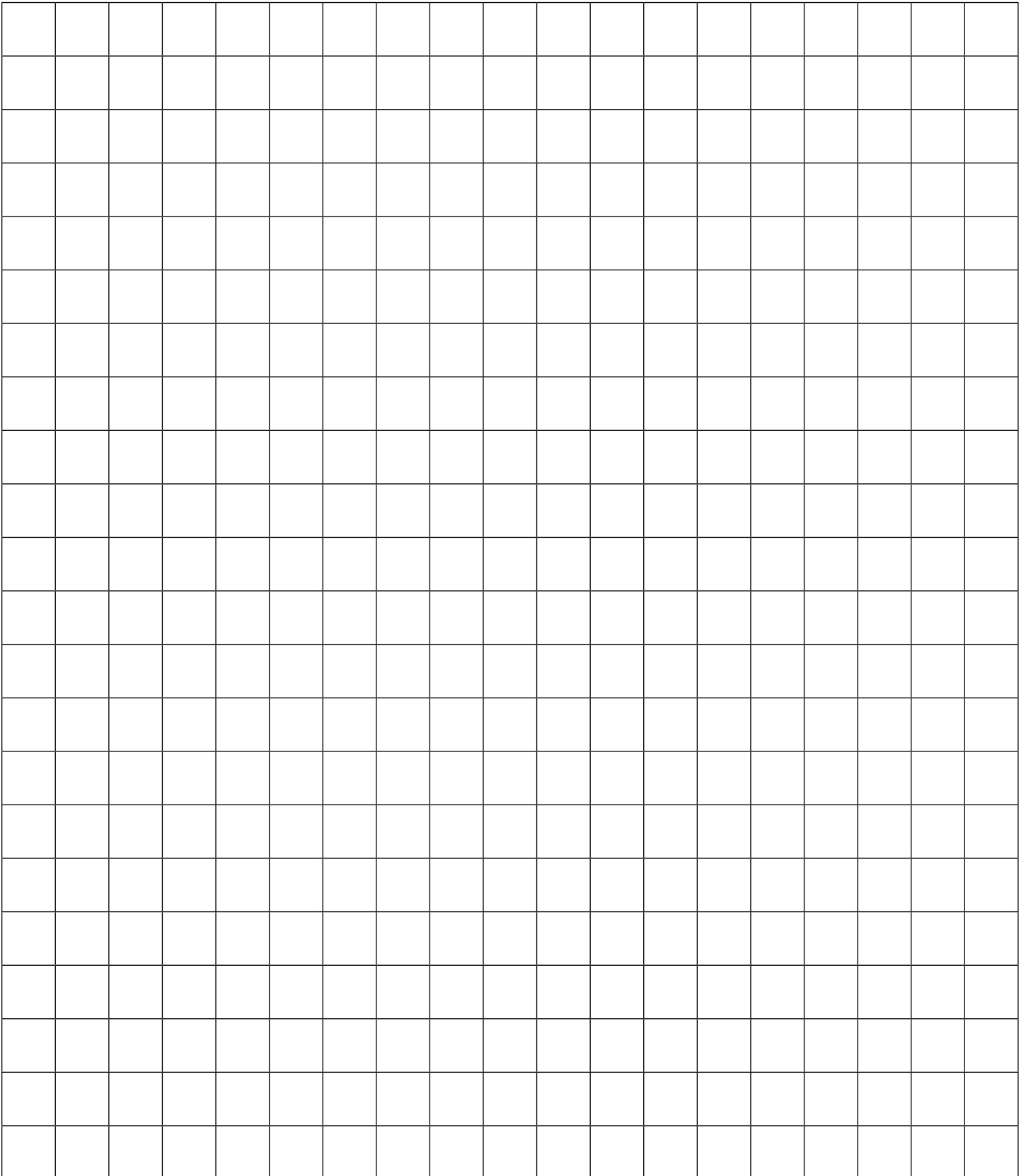
Key models for building connections extending to tool use



Key models for building connections the area model



Key models for building connections the area model



Key models for building connections the area model

Focus on Factors

60	70	90	120	140	160
70	90	120	140	160	180
60	450	480	600	180	210
800	420	80	700	210	270
700	360	350	300	270	300
600	480	450	420	360	350

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Key models for building connections the area model

Focus on Factors

Odd Numbers

5	5	15 (3 x 5)	25 (5 x 5)
5	5	15 (3 x 5)	25 (5 x 5)

Even Numbers

12 (2 x 6) (3 x 4)	14 (2 x 7)	24 (2 x 12) (3 x 8) (4 x 6)	32 (2 x 16) (4 x 8)
12 (2 x 6) (3 x 4)	14 (2 x 7)	24 (2 x 12) (3 x 8) (4 x 6)	32 (2 x 16) (4 x 8)

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Enlarge to make cards more manageable.

Key models for dividing numbers

Quotitive Division

1. $9 \overline{)257}$

2. $15 \overline{)1325}$

3. $22 \overline{)2435}$

4. $44 \overline{)2435}$

Key models for dividing numbers

Quotitive Division

1. $1\frac{1}{2} \div \frac{1}{2} =$

2. $1\frac{2}{3} \div \frac{1}{6} =$

3. $3\frac{1}{2} \div \frac{1}{8} =$

4. $2\frac{2}{3} \div \frac{2}{3} =$

5. $1\frac{1}{6} \div \frac{1}{3} =$

6. $3\frac{1}{6} \div \frac{1}{2} =$

Key models for dividing numbers

Quotitive Division

1. $1\frac{1}{2} \div \frac{1}{10} =$

2. $1\frac{1}{2} \div .1 =$

3. $1.5 \div .1 =$

4. $2.3 \div .1 =$

5. $3.4 \div .01 =$

6. $2.23 \div .01 =$

Building a conceptual web

