Multiplication Isn't Always Commutative: Exploring the Problems with Problem Solving

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1

Commutative or Not Commutative?

1. Putting on your coat and putting on your boots? ⇄
2. Washing your clothes and drying them?
3. Putting on your left shoe and your right shoe? ⇄
4. Hanging up the phone and saying good-bye? ⇄
5. Sautéeing vegetables and cutting the vegetables?
6. Turning on car bluetooth and phone bluetooth to pair?

2

Write a Word Problem

Represent your problem using counters and pictures. Choose ***either*** expression.

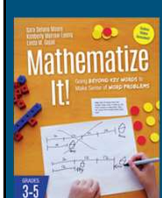
2×8	8×2

**If you have time, work with the other arrangement.*

3

Operation Sense

Knowing and applying the full range of work for mathematical operations (for example, addition, subtraction, multiplication, and division).



Number sense and operation sense are separate but complementary ideas.

4

Operation Sense

- **Working Models of Operations**
  - Understand and use a wide variety of models of operations beyond the basic and intuitive models of operations.
- **Representations of Operations**
  - Use appropriate representations of actions or relationships strategically.
- **Mathematizing**
  - Can mathematize a situation, translating a contextual understanding into a variety of other mathematical representations.
- **Number Categories**
  - Apply their understanding of operations to any quantity, regardless of the class of number.

5

This is not about computation.

Computational Strategies

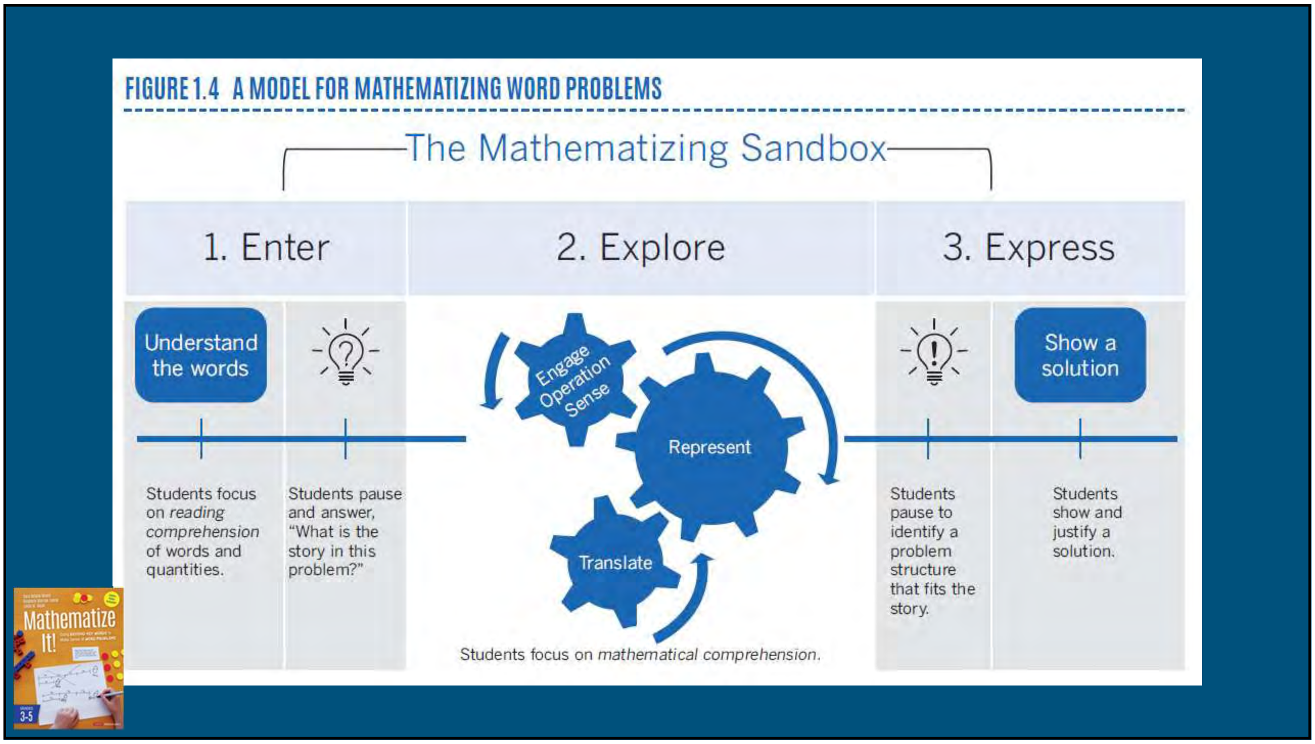
- Counting on or back
- Doubles
- Bridging ten
- Known related facts

- These are strategies students use to compute the answer, AFTER students understand the situation.

Models of Operations

- Operation Sense is about describing what is happening in the situation. What models and representations show the action or relationships in the problem?
- Operation sense comes BEFORE students select a computation strategy to find the solution.

6



7

What kind of multiplication are these?

Equal Groups (vases)

- Factors have different jobs
  - How many groups?
  - How large is each group?
- Exchanging quantities changes the meaning.

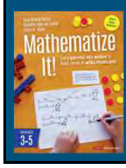
Area/Array (rug)

- Factors have the same job
- Product has a new unit of measure
- Exchanging quantities typically does not change meaning.

8

Multiplication and Division Problem Situations

ASYMMETRICAL (NONMATCHING) FACTORS				
	Product Unknown	Multiplier (Number of Groups) Unknown	Measure (Group Size) Unknown	
Equal Groups (Ratio/Rate)*	Mayim has 8 vases to decorate the tables at her party. She places 3 flowers in each vase. How many flowers does she need? $8 \times 3 = x$ $x = 8 \times 3$	Mayim has some vases to decorate the tables at her party. She places 3 flowers in each vase. If she uses 24 flowers, how many vases does she have? $x \times 3 = 24$ $x = 24 \div 3$	Mayim places 24 flowers in vases to decorate the tables at her party. If there are 8 vases, how many flowers will be in each vase? $8 \times x = 24$ $24 \div 8 = x$	
	Resulting Value Unknown	Scale Factor (Times as Many) Unknown	Original Value Unknown	
Multiplicative Comparison	Amelia's dog is 5 times older than Wanda's 3-year-old dog. How old is Amelia's dog? $5 \times 3 = x$ $x = 5 \times 3$	Sydney has $15 to spend on dog treats. Her best friend has $5. Sydney has how many times more dollars than her friend has? $x \times 5 = 15$ $x = 15 \div 5$	Devonte has 15 dog toys on the floor in his living room. That is 3 times the number of toys in the dog's toy basket. How many toys are in the toy basket? $3 \times x = 15$ $15 \div 3 = x$	
SYMMETRICAL (MATCHING) FACTORS				
	Product Unknown	One Dimension Unknown		Both Dimensions Unknown
Area/Array	Mrs. Bradley bought a rubber mat to cover the floor under the balance beam. One side of the mat measured 5 feet and the other side measured 8 feet. How many square feet does the mat measure? $5 \times 8 = x$ $x = 8 \times 5$	The 40 members of the student council lined up on the stage in the gym to take yearbook pictures. The first row started with 8 students and the rest of the rows did the same. How many rows were there? $8 \times x = 40$ $x = 40 \div 8$		Mr. Donato is arranging student work on the wall for the art show. He started with 40 square entries and arranged them into a rectangular arrangement. How many entries long and wide could the arrangement be? $x \times y = 40$ $40 \div x = y$
	Sample Space (Total Outcomes) Unknown	One Factor Unknown		Both Factors Unknown
Combinations** (Fundamental Counting Principle)	Karen makes sandwiches at the diner. She offers 3 kinds of bread and 7 different lunch meats. How many unique sandwiches can she make? $3 \times 7 = x$ $3 \times x = 7$	Evelyn works at the ice cream counter. She says that she can make 21 unique and different ice cream sundaes using just ice cream flavors and toppings. If she has 3 flavors of ice cream, how many kinds of toppings does Evelyn offer? $3 \times x = 21$ $21 \div 3 = x$		Audrey can make 21 different fruit sodas using the machine at the diner. How many different flavorings and sodas could there be? $x \times y = 21$ $x = 21 \div y$



9

Evidence from Research: Japan

U.S. Textbooks

- Investigate the commutative property *near the beginning* of instruction of facts.
- Treat the commutative property as *obvious*.
- Does not distinguish multiplier factor (how many groups?) from measure factor (number in a group).

Japanese Textbooks

- Commutative property is introduced *after* all facts are introduced.
- Do *not* assume the property is obvious.
- Distinguishes the multiplier factor (how many groups?) from the measure factor (number in a group).

Watanabe, T. (2003). Teaching multiplication: An analysis of elementary school mathematics teachers' manuals from Japan and the United States. *The Elementary School Journal*, *104*(2), 111–125. https://doi.org/10.1086/499745.

10

Evidence from Research: Africa (Ivory Coast)

United States

- U.S. college students *frequently* used the commutative property.
- Only *one* U.S. student used the associative property
- Relied on algorithms.

Ivory Coast (Dioula)

- Unschooled Dioula young adults *rarely* used the commutative property
- Unschooled Dioula young adults *frequently* used the associative property
- Relied on repeated addition.

Petitto, A. L., & Ginsburg, H. P. (1982). Mental arithmetic in Africa and America: Strategies, principles, and explanations. *International Journal of Psychology*, *17*(1–4), 81–102. https://doi.org/10.1080/00207598208247433.

11

Guidelines for using the Commutative Property

1. Be mindful of moving out of context to use the commutative property for computation. It may be nonsensical to children.
2. Equal Groups is not the best problem type for teaching the commutative property.
3. Let the numbers get big (beyond known facts) before you start to focus on the commutative property. Let it be a GIFT! The commutative property *should solve a problem for students.*
4. Write the expression for the problem first (solution), then talk about computation strategies for getting the answer - including (perhaps) the commutative property.

12

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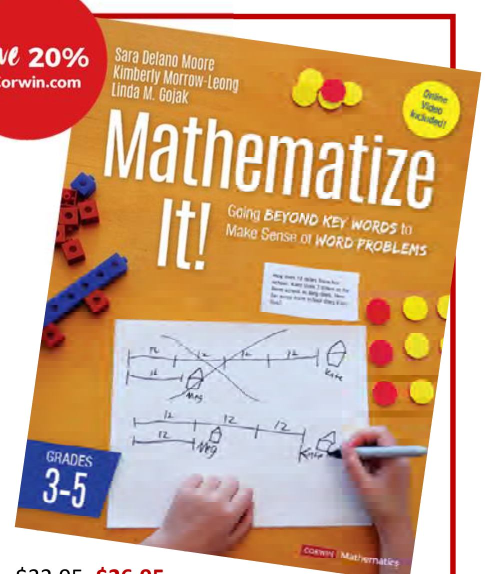

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14