4.11 Common fractions: Representing as a sum of unit fractions

In this lesson, students use number lines to explore how common fractions are composed of unit fractions. Students consider what the numerator and denominator mean in this context.

Step 1 Preparing the lesson

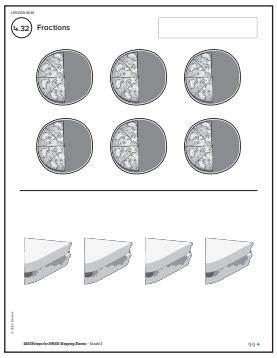
You will need:

- counters or cubes

Each student will need:

- 1 copy of Blackline Master 4.32
- Student Journal 4.11

Blackline Master 4.32



Step 2 Starting the lesson

Ask students to describe what it means to skip count and have them share what numbers they are able to skip count. Emphasize that if they are counting by twos, for example, that each number in the sequence represents the result of adding two repeatedly. Use counters or cubes to support the concept.

Step 3 Teaching the lesson

Draw a number line from 0 to 25 on the board. It should show five jumps labeled +5 along the length. Ask, *What does this number line show? What do the numbers above the number line mean?* (They show that 5 is added repeatedly.) *What do the numbers below the number line mean?* (They show the result of adding 5 repeatedly.) Emphasize that they are the numbers that are said when skip counting by fives.

Distribute Blackline Master 4.32. Direct students to the first picture of half pizzas and have them count the number of halves with you. Point to each half as you count (one half, two halves, etc). Say, *There are 6 halves of pizza in the picture.* Then direct students to the second picture of fourths of a sandwich and say, *These are from a sandwich that was cut into fourths. Let us count the number of fourths.* Have the students count and confirm that there are 4 fourths in the picture.

Draw a number line sectioned into fourths (unlabeled) on that board, and say, *We are going to skip count by unit fractions*. Explain that when common fractions have 1 as the numerator it is called a unit fraction. Say, *This number line will keep track of the fractions as we skip count* (**SMP7**). Draw an arrow that shows the jump from 0 to $\frac{1}{4}$. Label the jump + $\frac{1}{4}$, then discuss the points below:

What number should we write below the number line? ($\frac{1}{4}$.)

If we make another jump of $\frac{1}{4}$, what number should we write below? ($\frac{2}{4}$.)

How many jumps do you need to make to reach one whole? (4.)

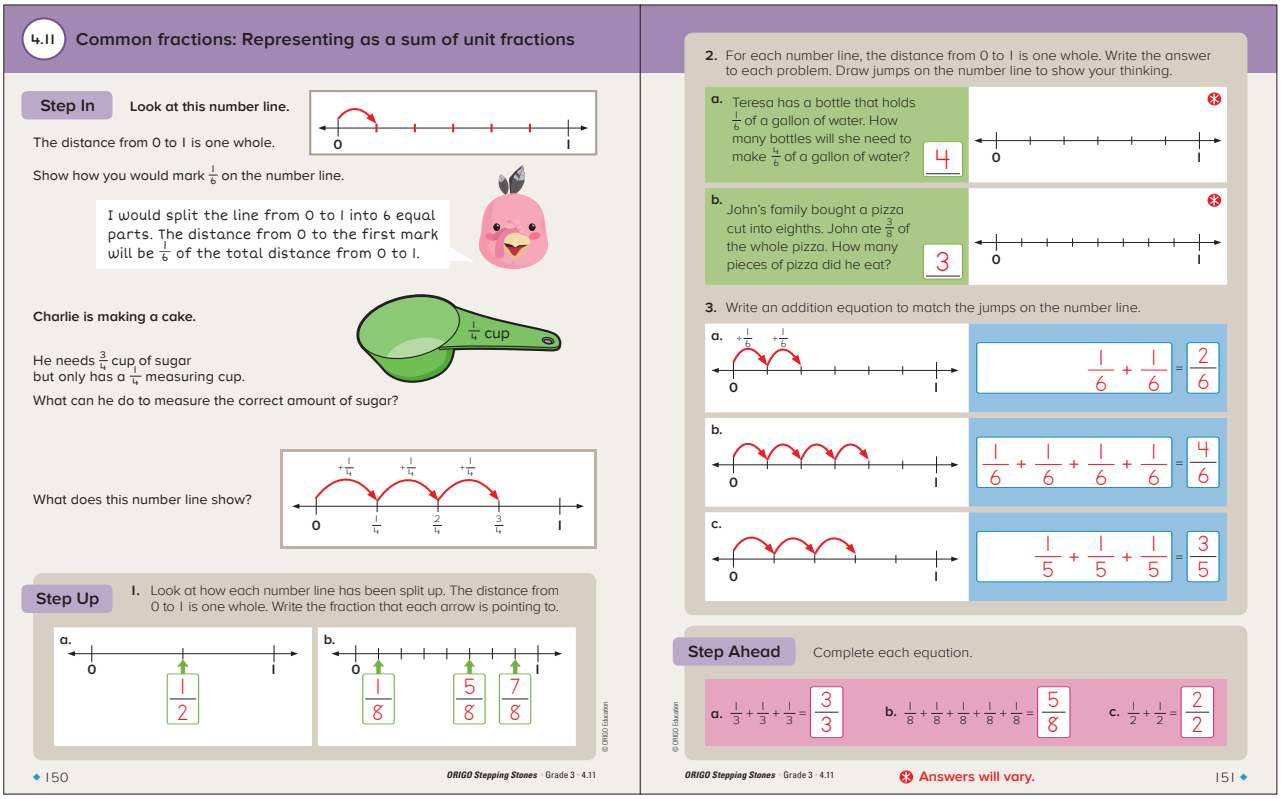
What equation could you write to match the jumps that were made? ($\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4}$.)

Draw and label the next jump and say, *2 fourths*. (*Note:* Some students may recognize that that $\frac{2}{4}$ is the same value as $\frac{1}{2}$ but the emphasis should be on $\frac{2}{4}$ and to label it accordingly.) Draw and label jumps until $\frac{4}{4}$ is written below the line. Establish that 4 fourths are the same value as 1 whole and write 1 whole below $\frac{4}{4}$ on the number line. Adjust the number line and repeat the activity to discuss thirds, then sixths.

ELL

Demonstrate the words *above*, *below*, and *diagonal*. Have the students discuss the words *halve* and *halves* before moving on with the activity. Create an anchor chart showing a unit fraction.

Student Journal 4.11, pp. 150–151



4.11 Common fractions: Representing as a sum of unit fractions

Step In Look at this number line.

The distance from 0 to 1 is one whole.

Show how you would mark $\frac{1}{6}$ on the number line.

I would split the line from 0 to 1 into 6 equal parts. The distance from 0 to the first mark will be $\frac{1}{6}$ of the total distance from 0 to 1.

Charlie is making a cake.

He needs $\frac{3}{4}$ cup of sugar but only has a $\frac{1}{4}$ measuring cup.

What can he do to measure the correct amount of sugar?

$\frac{1}{4}$ cup

What does this number line show?

Step Up

1. Look at how each number line has been split up. The distance from 0 to 1 is one whole. Write the fraction that each arrow is pointing to.

a. $\frac{1}{2}$

b. $\frac{1}{8}$ $\frac{5}{8}$ $\frac{7}{8}$

150 ORIGO Stepping Stones · Grade 3 · 4.11

2. For each number line, the distance from 0 to 1 is one whole. Write the answer to each problem. Draw jumps on the number line to show your thinking.

a. Teresa has a bottle that holds $\frac{1}{6}$ of a gallon of water. How many bottles will she need to make $\frac{4}{6}$ of a gallon of water? 4

b. John's family bought a pizza cut into eighths. John ate $\frac{3}{8}$ of the whole pizza. How many pieces of pizza did he eat? 3

3. Write an addition equation to match the jumps on the number line.

a. $\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$

b. $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{4}{6}$

c. $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$

Step Ahead Complete each equation.

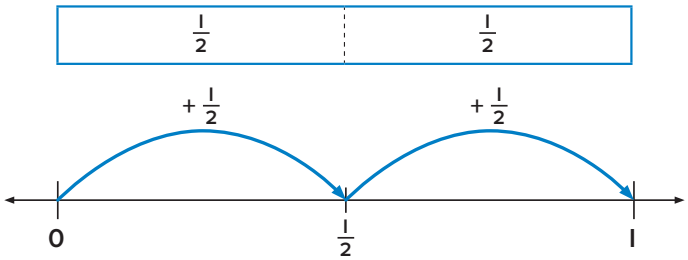
a. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3}$ b. $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{5}{8}$ c. $\frac{1}{2} + \frac{1}{2} = \frac{2}{2}$

ORIGO Stepping Stones · Grade 3 · 4.11 Answers will vary. 151

Work through the Step In discussion (Student Journal 4.11) with the whole class. Read the Step Up and Step Ahead instructions with the students. Make sure they know what to do, then have them work independently to complete the tasks. As you walk around the room have students describe how they figured out the size of the jumps in Question 3.

Step 4 Reflecting on the work

Discuss the students' answers to Student Journal 4.11. Draw the diagrams, as shown below, on the board and emphasize that the models show different things. The fraction strip model compares the length of each part to the length of the whole strip. The number line model records the number that results from counting by fractions. Say, *On the number line, $\frac{1}{2}$ is a number that is labeled in only one place below the number line* (**SMP6**). *The number $\frac{1}{2}$ is half the distance from 0 to 1 on the number line.* Adjust the number line and strip diagrams to compare representations of one-third and repeat the discussion.



Applications

If time allows, have the students complete this Investigation and/or Problem solving activity.

Investigation: Working with unit fractions

Each group of four students will need:

- 1 cube labeled: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$

Organize the students into groups of four and distribute the cubes. Say, *Imagine you are playing a game. You roll the fraction cube and record the fraction. You write tallies to record the number of times you roll each fraction.* Write the investigation question, as shown, on the board and read it with the students. To ensure understanding of the task, ask questions such as, *How many times would you need to roll $\frac{1}{4}$ to make one whole?* (4 times.) *How many times would you need to roll $\frac{1}{8}$?* Organize students into pairs to complete the investigation. Afterward, invite students to share their findings and explain their methods.

Investigation question

How many times will you need to roll the fraction cube before you can make one whole with fractions that have the same denominator?

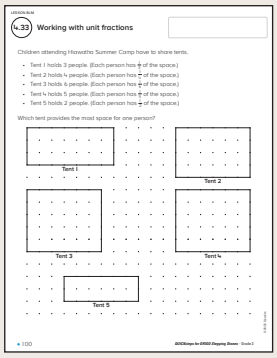
Problem solving: Working with unit fractions

Each pair of students will need:

- 1 copy of Blackline Master 4.33

Organize the students into pairs and distribute the blackline master. Read the word problem with the class. Ask questions such as, *How can you determine how much space each person has for each tent? Are the tents the same size? Can we rely on the size of the fraction to determine which tent would have more space?* Discuss the sizes of the tents. Highlight how they are not all the same and that affects the amount of space available for each person. Highlight how $\frac{1}{2}$ of Tent 5 is actually less space than $\frac{1}{3}$ of Tent 1, even though $\frac{1}{2}$ is more than $\frac{1}{3}$ on a number line. Encourage students to think about the amount of square units each person would have. Have them work together to determine which tent provides the most space for one person. Afterward, invite pairs to share their solution and explain their strategies (Tent 1 provides the greatest amount of space: 7 square units of space for each person).

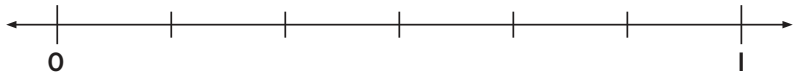
Blackline Master 4.33



Small group differentiation

Extra help

Draw the number line, as shown, on the board (or use the *Flare Fractions (Length)* online tool) and have a student confirm that the number line represents sixths. Then have the students count in sixths as you draw the matching number of jumps above the number line. Repeat with other fractions.



Extra practice

Each group of students will need:

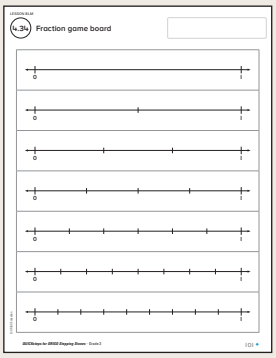
- 1 cube labeled: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{9}$

Each student will need:

- 1 copy of Blackline Master 4.34

Organize the students into groups and distribute the resources. They take turns to roll the cube. They then draw one jump above a number line on their game board to match the fraction rolled. The first student to reach one whole on two different number lines wins the game.

Blackline Master 4.34



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Step In

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Show how you would mark $\frac{1}{6}$ on the number line.

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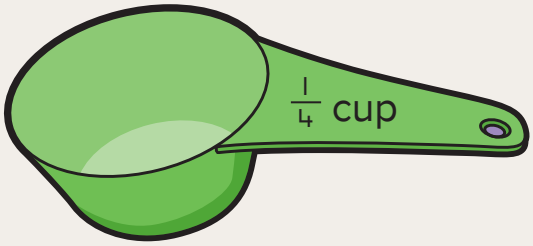
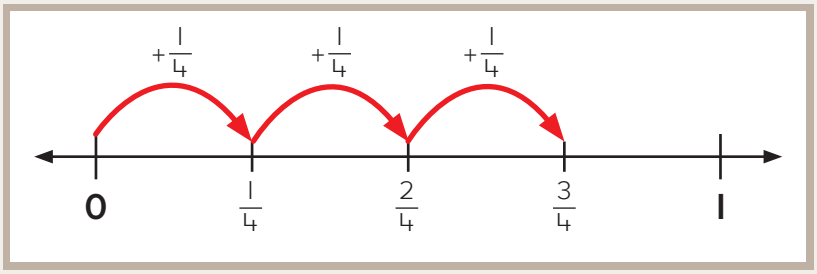
Charlie is making a cake.



He needs $\frac{3}{4}$ cup of sugar
but only has a $\frac{1}{4}$ measuring cup.

What can he do to measure the correct amount of sugar?

What does this number line show?



Step Up

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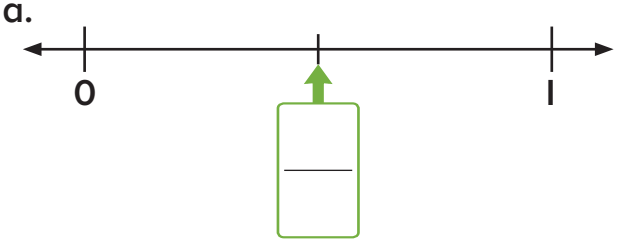
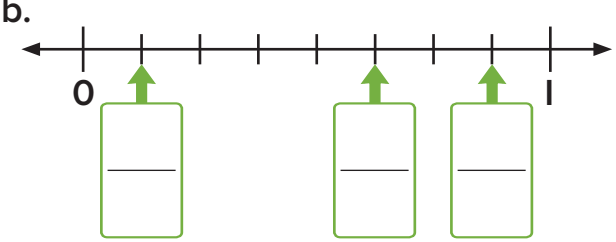
a.



b.



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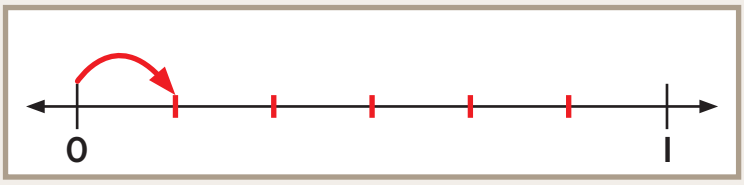

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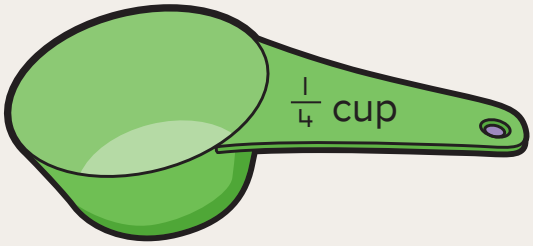

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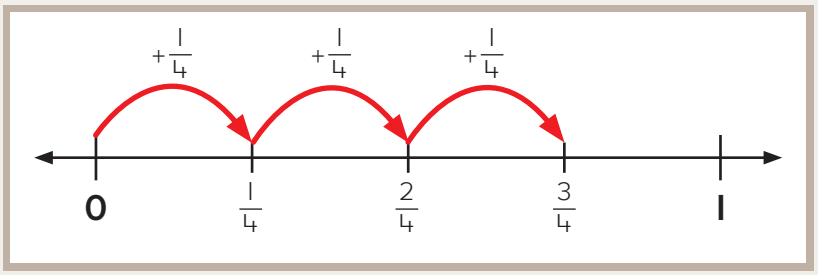

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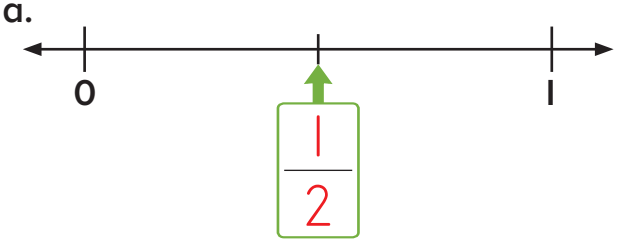

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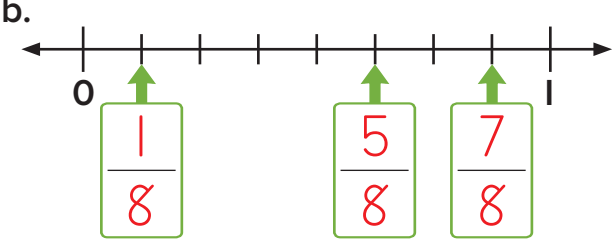

2. For each number line, the distance from 0 to 1 is one whole. Write the answer to each problem. Draw jumps on the number line to show your thinking.

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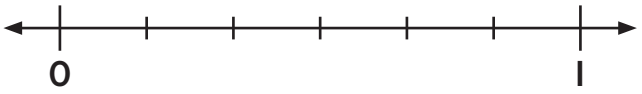
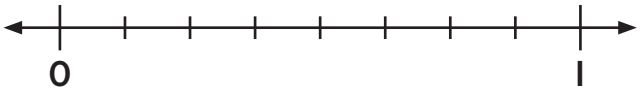
_____



b. John's family bought a pizza cut into eighths. John ate $\frac{3}{8}$ of the whole pizza. How many pieces of pizza did he eat?

_____



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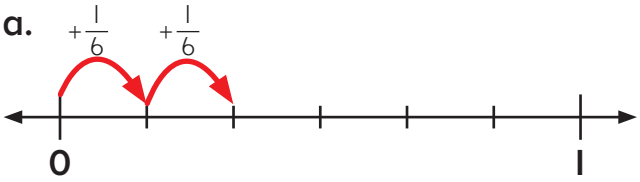
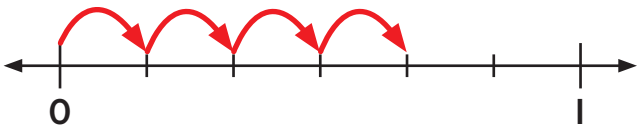
a.



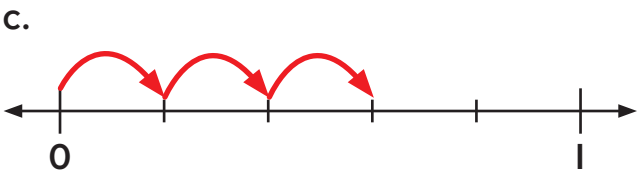
_____ = _____

b.



_____ = _____

c.



_____ = _____

Step Ahead

Complete each equation.

a. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{\quad}{3}$

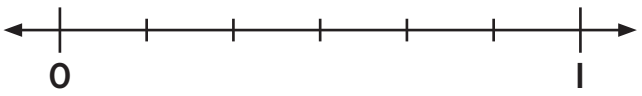
b. $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{\quad}{8}$

c. $\frac{1}{2} + \frac{1}{2} = \frac{\quad}{2}$

2. For each number line, the distance from 0 to 1 is one whole. Write the answer to each problem. Draw jumps on the number line to show your thinking.

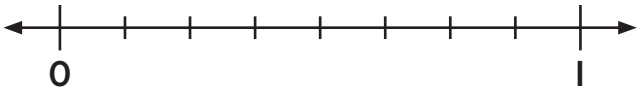
a. Teresa has a bottle that holds $\frac{1}{6}$ of a gallon of water. How many bottles will she need to make $\frac{4}{6}$ of a gallon of water?

4



b. John's family bought a pizza cut into eighths. John ate $\frac{3}{8}$ of the whole pizza. How many pieces of pizza did he eat?

3



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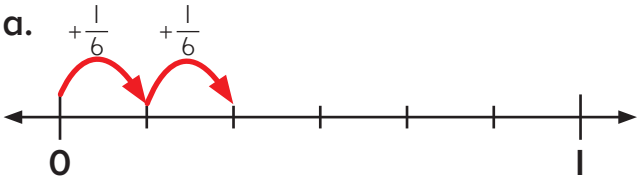
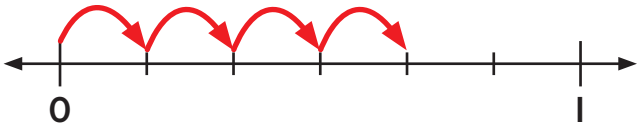


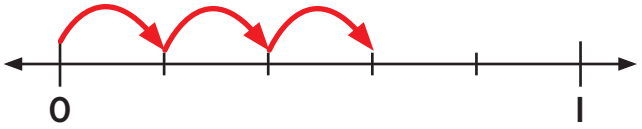
$\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$

b.



$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{4}{6}$

c.



$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$

Step Ahead

Complete each equation.

a. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3}$

b. $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{5}{8}$

c. $\frac{1}{2} + \frac{1}{2} = \frac{2}{2}$

Answers will vary.

4.11 Fracciones comunes: Representándolas como la suma de fracciones unitarias

Conoce

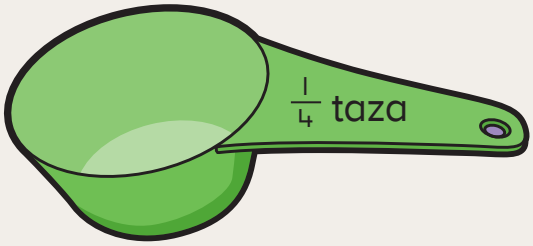
Observa esta recta numérica.

La distancia de 0 a 1 es un entero.

Indica cómo marcarías $\frac{1}{6}$ en la recta numérica.

Yo partiría la recta numérica de 0 a 1 en 6 partes iguales. La distancia de 0 a la primera marca será $\frac{1}{6}$ de la distancia total de 0 a 1.

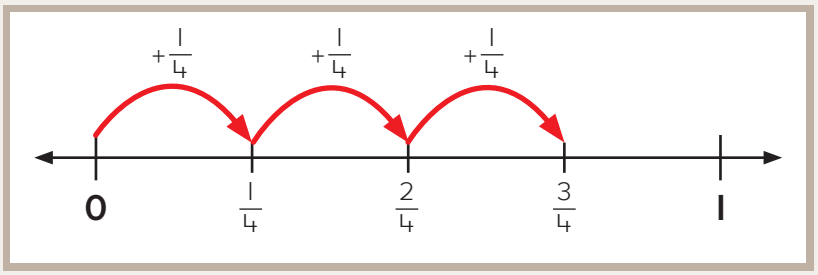
Charlie va a hacer un pastel.



Él necesita $\frac{3}{4}$ de taza de azúcar pero solo tiene una taza de medida de $\frac{1}{4}$.

¿Qué puede hacer él para medir la cantidad correcta de azúcar?

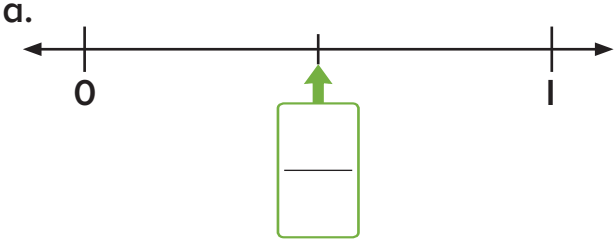
¿Qué indica esta recta numérica?



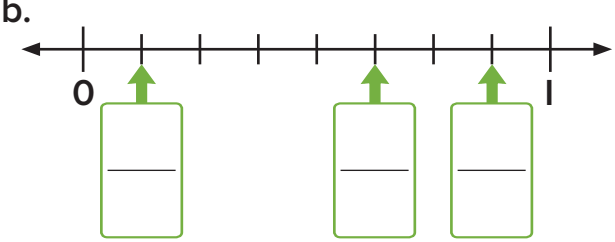
Intensifica

1. Observa cómo se ha partido cada recta numérica. La distancia de 0 a 1 es un entero. Escribe la fracción a la que apunta cada flecha.

a.



b.



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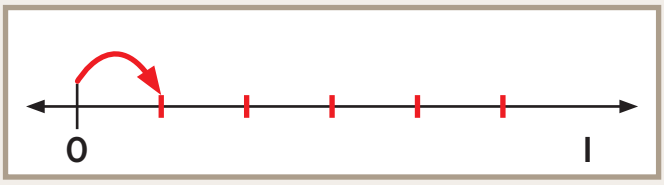

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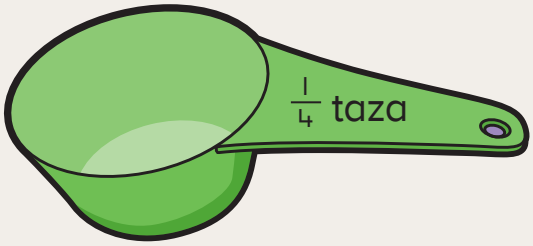

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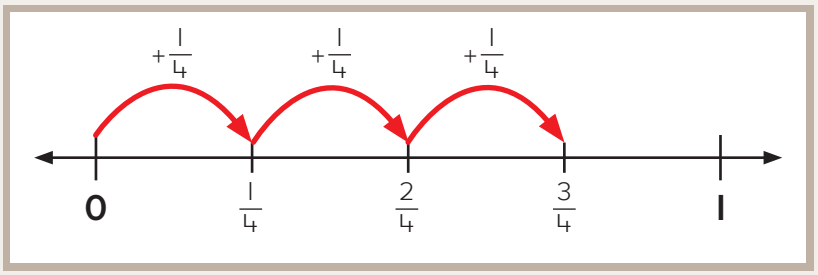
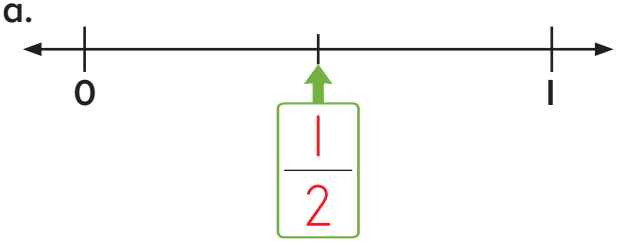


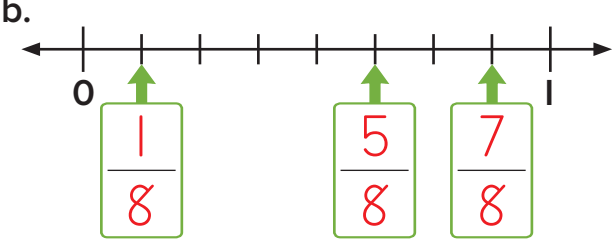
Intensifica

1. Observa cómo se ha partido cada recta numérica. La distancia de 0 a 1 es un entero. Escribe la fracción a la que apunta cada flecha.

a.



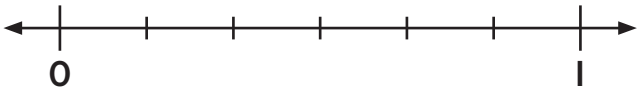
b.



2. En cada recta numérica la distancia de 0 a 1 es un entero. Escribe la respuesta a cada problema. Dibuja saltos en la recta para indicar tu razonamiento.

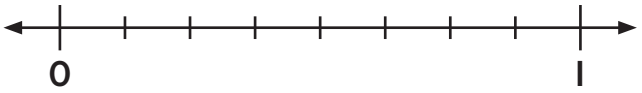
a. Teresa tiene una botella que contiene $\frac{1}{6}$ de galón de agua. ¿Cuántas botellas necesitará para tener $\frac{4}{6}$ de galón de agua?

_____



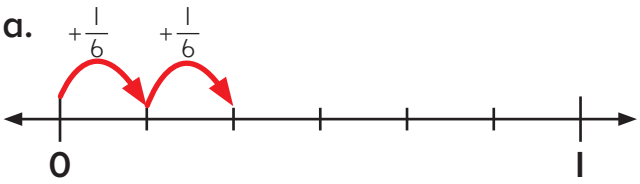
b. La familia de John compró una pizza cortada en octavos. John se comió $\frac{3}{8}$ de la pizza. ¿Cuántos trozos de pizza se comió?

_____



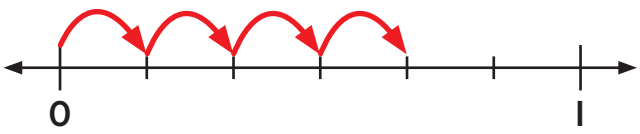
3. Escribe una ecuación de suma que corresponda a los saltos en la recta numérica.

a.



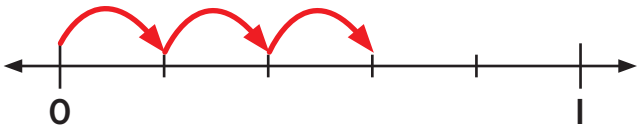
_____ = ____

b.



_____ = ____

c.



_____ = ____

Avanza

Completa cada ecuación.

a. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{\quad}{3}$

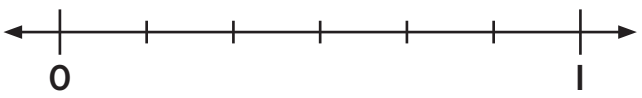
b. $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{\quad}{8}$

c. $\frac{1}{2} + \frac{1}{2} = \frac{\quad}{2}$

2. En cada recta numérica la distancia de 0 a 1 es un entero. Escribe la respuesta a cada problema. Dibuja saltos en la recta para indicar tu razonamiento.

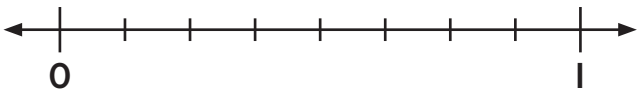
a. Teresa tiene una botella que contiene $\frac{1}{6}$ de galón de agua. ¿Cuántas botellas necesitará para tener $\frac{4}{6}$ de galón de agua?

4



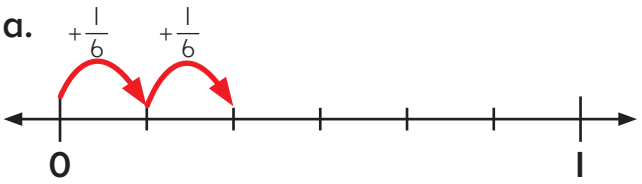
b. La familia de John compró una pizza cortada en octavos. John se comió $\frac{3}{8}$ de la pizza. ¿Cuántos trozos de pizza se comió?

3



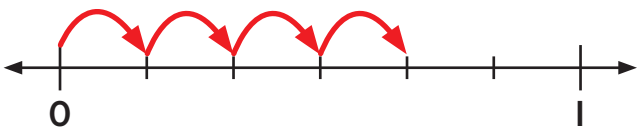
3. Escribe una ecuación de suma que corresponda a los saltos en la recta numérica.

a.



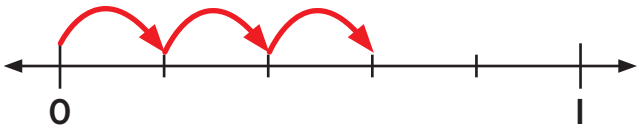
$\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$

b.



$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{4}{6}$

c.



$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$

Avanza

Completa cada ecuación.

a. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3}$

b. $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{5}{8}$

c. $\frac{1}{2} + \frac{1}{2} = \frac{2}{2}$

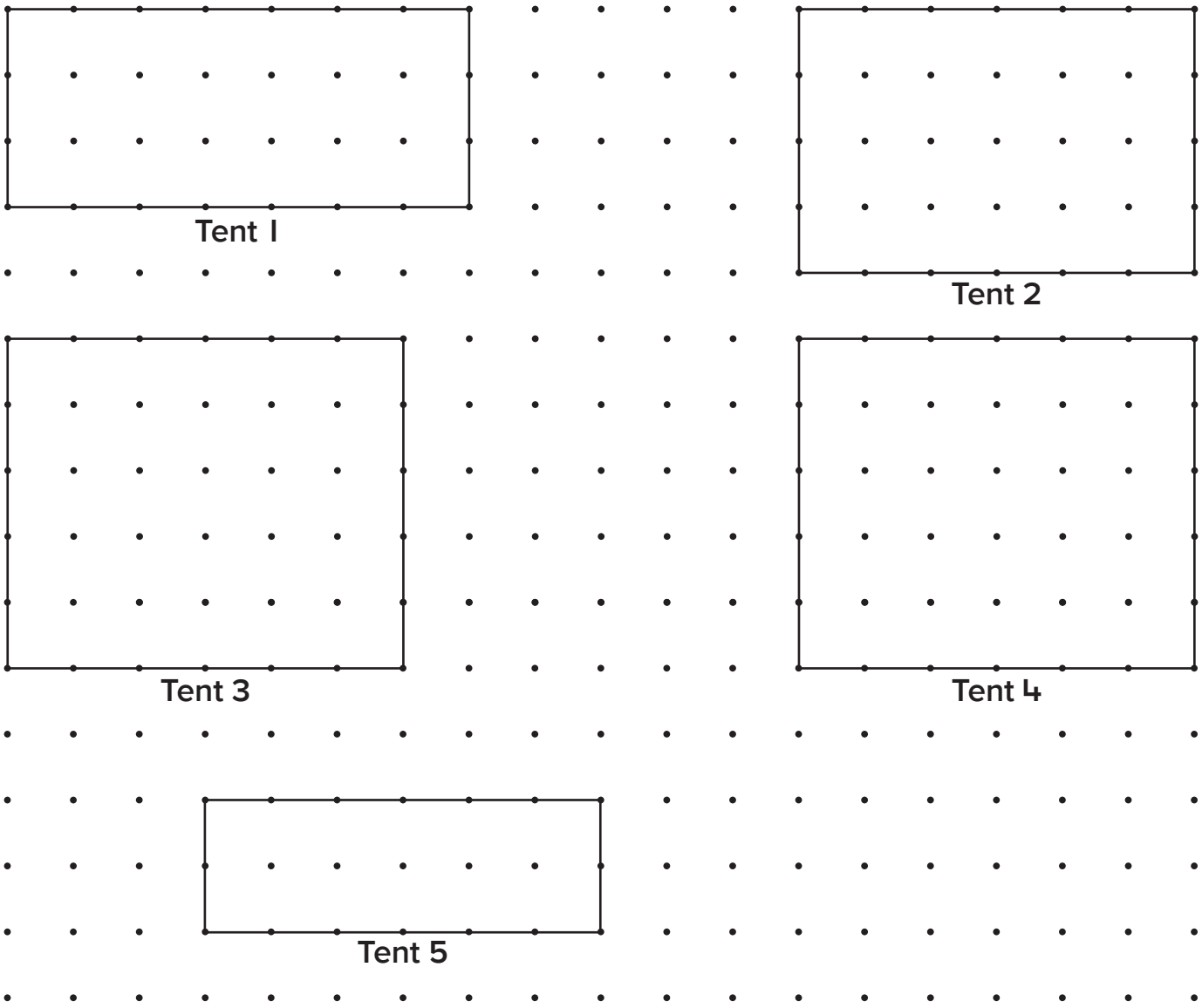
Las respuestas pueden variar.

4.33 Working with unit fractions

Children attending Hiawatha Summer Camp have to share tents.

- Tent 1 holds 3 people. (Each person has $\frac{1}{3}$ of the space.)
- Tent 2 holds 4 people. (Each person has $\frac{1}{4}$ of the space.)
- Tent 3 holds 6 people. (Each person has $\frac{1}{6}$ of the space.)
- Tent 4 holds 5 people. (Each person has $\frac{1}{5}$ of the space.)
- Tent 5 holds 2 people. (Each person has $\frac{1}{2}$ of the space.)

Which tent provides the most space for one person?

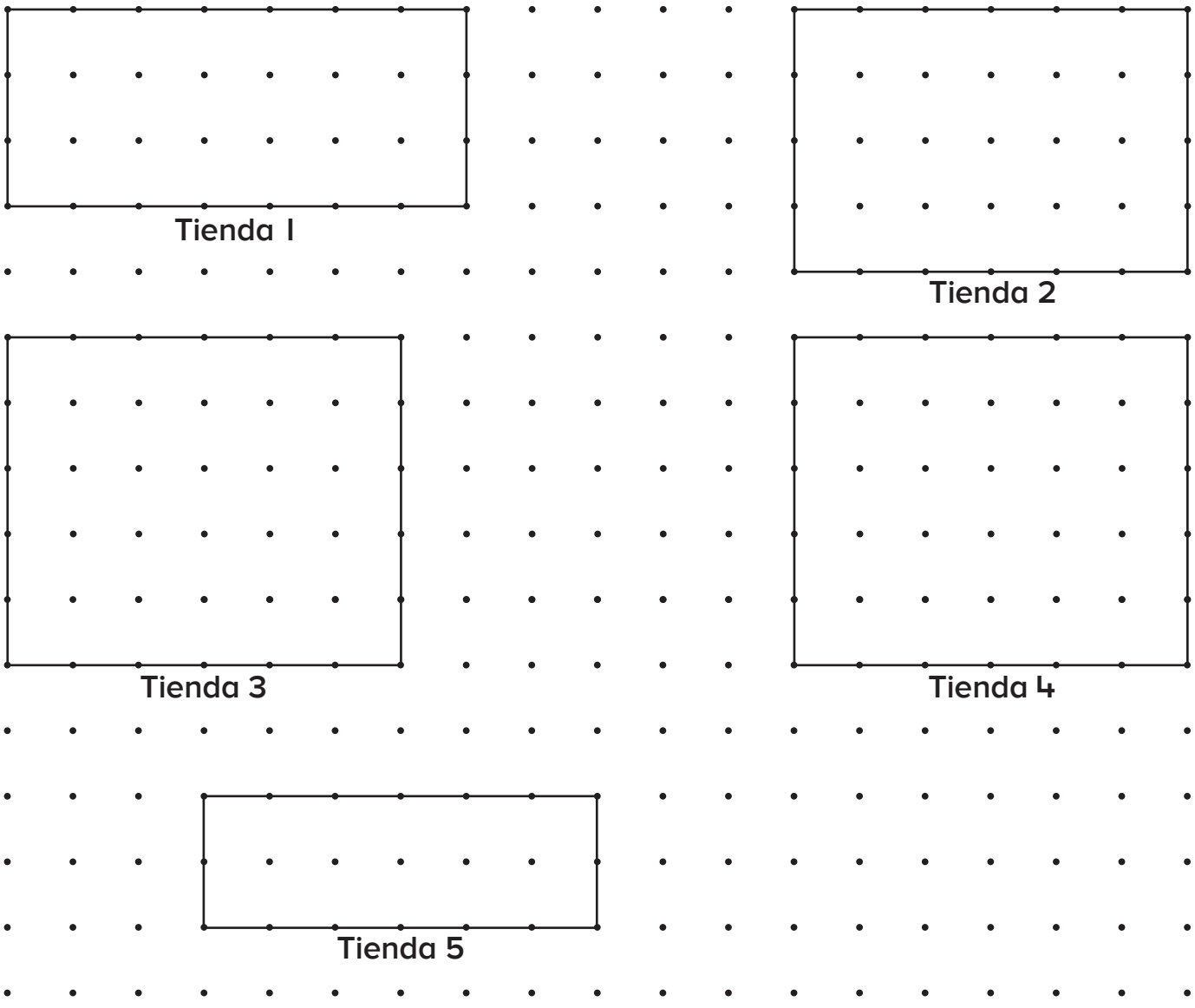


4.33 Trabajando con fracciones unitarias

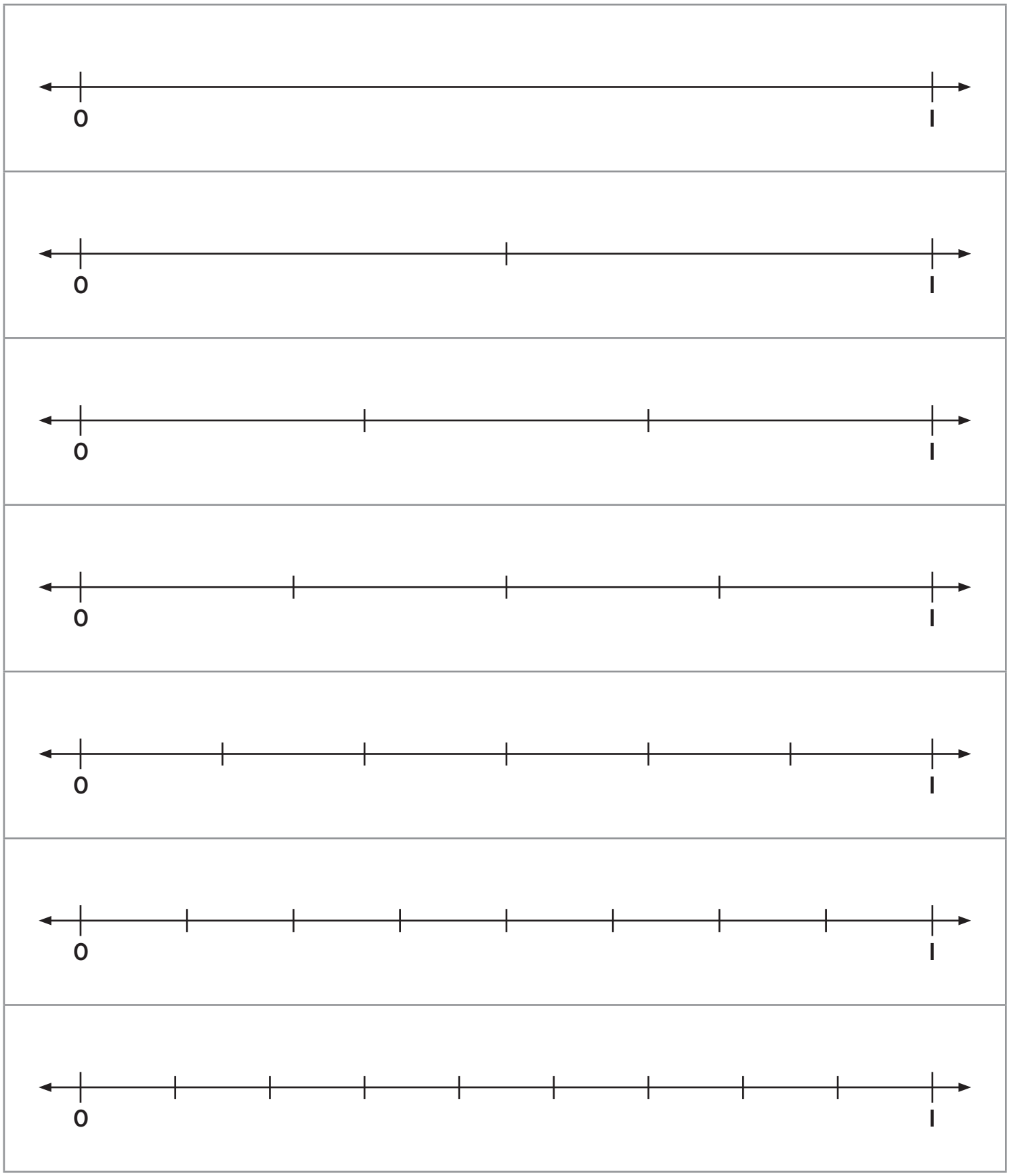
Los niños que asistirán al campamento de verano de Hiawatha tienen que compartir tiendas de campaña.

- En la tienda 1, caben 3 personas. (Cada persona tiene $\frac{1}{3}$ del espacio).
- En la tienda 2, caben 4 personas. (Cada persona tiene $\frac{1}{4}$ del espacio).
- En la tienda 3, caben 6 personas. (Cada persona tiene $\frac{1}{6}$ del espacio).
- En la tienda 4, caben 5 personas. (Cada persona tiene $\frac{1}{5}$ del espacio).
- En la tienda 5, caben 2 personas. (Cada persona tiene $\frac{1}{2}$ del espacio).

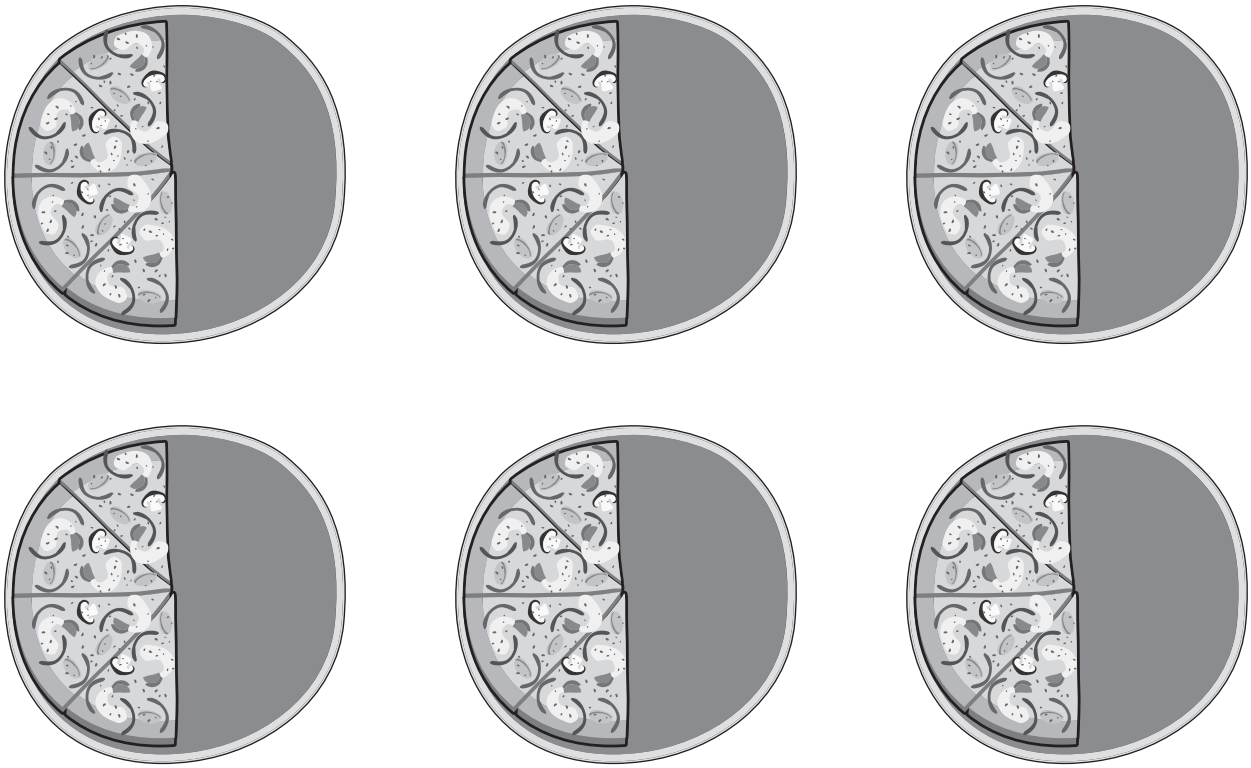
¿Cuál de las tiendas de campaña ofrece el mayor espacio para una persona?

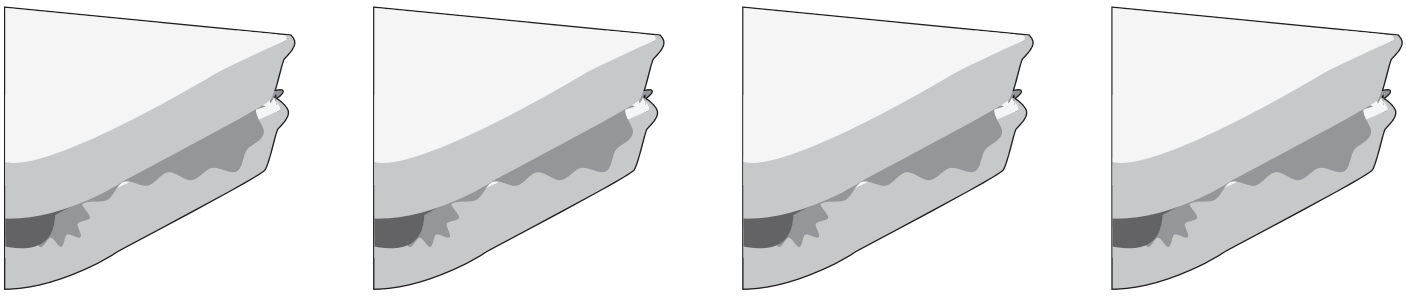


4.34 Fraction game board



4.32 Fractions





3 Working with unit fractions

Children attending Hiawatha Summer Camp have to share tents.

- Tent 1 holds 3 people. (Each person has $\frac{1}{3}$ of the space.)
- Tent 2 holds 4 people. (Each person has $\frac{1}{4}$ of the space.)
- Tent 3 holds 6 people. (Each person has $\frac{1}{6}$ of the space.)
- Tent 4 holds 5 people. (Each person has $\frac{1}{5}$ of the space.)
- Tent 5 holds 2 people. (Each person has $\frac{1}{2}$ of the space.)

Which tent provides the most space for one person?