8.7 Common fractions: Identifying improper fractions on a number line

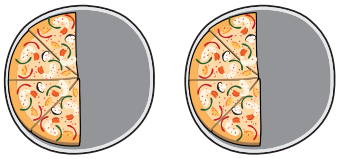
In this lesson, students use the number line to explore fractions that are equal to or greater than 1. Students consider the relationship between the numerator and denominator with improper fractions.

Step 1 Preparing the lesson

Each student will need:

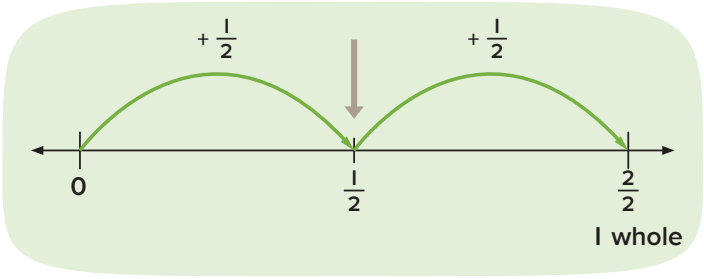
- paper
- Student Journal 8.7

Step 2 Starting the lesson



Draw two half pizzas, as shown, on the board. Count the number of half pizzas aloud with the class. Say, *We are going to record the halves on a number line. How many half pizzas were there?* (2.)

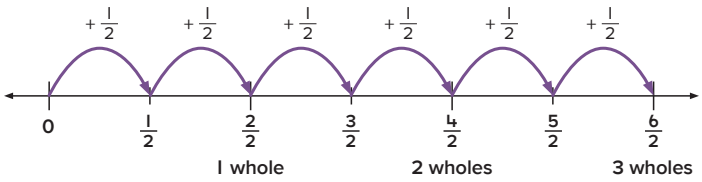
Draw an empty number line on the board. Have the students count aloud as you draw and label each jump of $+\frac{1}{2}$ above the number line. Ask, *What numbers should we write below the number line?* Record $\frac{1}{2}$ and $\frac{2}{2}$ below the line and ask, *What whole number is the same value as two halves?* (1 whole.) Write 1 whole below $\frac{2}{2}$ as shown below.



Step 3 Teaching the lesson

Draw six half pizzas on the board. Extend the number line on the board, and have the students continue to count the halves aloud as you draw and label each jump $+\frac{1}{2}$ above the number line, as shown below. Ask, *What numbers should we write below the number line?* Record $\frac{3}{2}$, $\frac{4}{2}$, $\frac{5}{2}$, and $\frac{6}{2}$ below the line and ask, *What whole number is the same value as four-halves? Six-halves?* (3 wholes.)

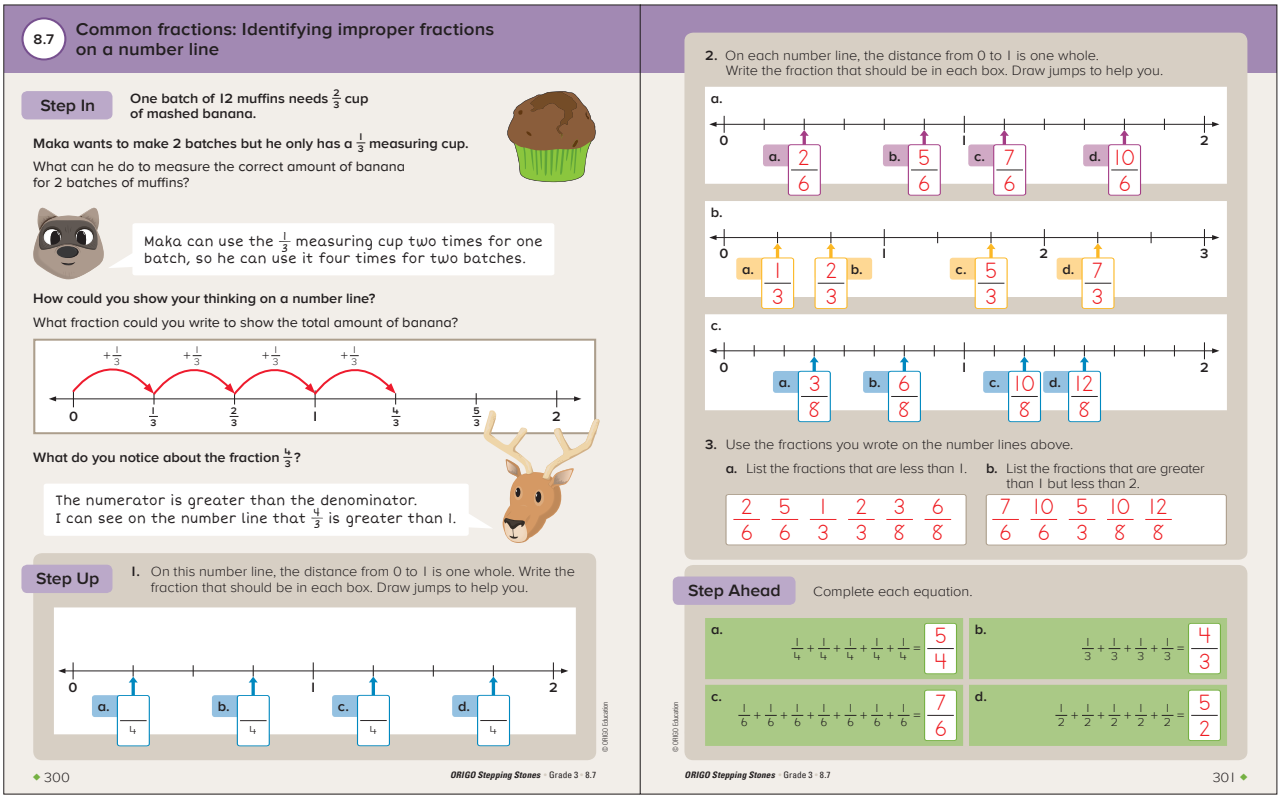
Write 2 wholes below $\frac{4}{2}$ and 3 wholes below $\frac{6}{2}$ on the number line, as shown below. Ask, *What do you notice about the numerators and denominators before and after one whole?* Guide students to explain that when the fractions are less than one whole, the numerator is less than the denominator, and when the fractions are greater than one whole, the numerator is greater than the denominator. Have students discuss why that makes sense (**SMP6**). Write the terms proper fraction and improper fraction below the number line on the board, to connect these terms with the position of the fractions on the number line.



ELL

Reinforce new language, such as *numerator* and *denominator*, by asking the students to use the words in a sentence.

Student Journal 8.7, pp. 300–301



8.7 Common fractions: Identifying improper fractions on a number line

Step In One batch of 12 muffins needs $\frac{2}{3}$ cup of mashed banana.

Maka wants to make 2 batches but he only has a $\frac{1}{3}$ measuring cup.
What can he do to measure the correct amount of banana for 2 batches of muffins?

Maka can use the $\frac{1}{3}$ measuring cup two times for one batch, so he can use it four times for two batches.

How could you show your thinking on a number line?
What fraction could you write to show the total amount of banana?

What do you notice about the fraction $\frac{4}{3}$?

The numerator is greater than the denominator.
I can see on the number line that $\frac{4}{3}$ is greater than 1.

Step Up 1. On this number line, the distance from 0 to 1 is one whole. Write the fraction that should be in each box. Draw jumps to help you.

2. On each number line, the distance from 0 to 1 is one whole. Write the fraction that should be in each box. Draw jumps to help you.

a. a. $\frac{2}{6}$ b. $\frac{5}{6}$ c. $\frac{7}{6}$ d. $\frac{10}{6}$

b. a. $\frac{1}{3}$ b. $\frac{2}{3}$ c. $\frac{5}{3}$ d. $\frac{7}{3}$

c. a. $\frac{3}{8}$ b. $\frac{6}{8}$ c. $\frac{10}{8}$ d. $\frac{12}{8}$

3. Use the fractions you wrote on the number lines above.

a. List the fractions that are less than 1. $\frac{2}{6}$ $\frac{5}{6}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{8}$ $\frac{6}{8}$

b. List the fractions that are greater than 1 but less than 2. $\frac{7}{6}$ $\frac{10}{6}$ $\frac{5}{3}$ $\frac{10}{8}$ $\frac{12}{8}$

Step Ahead Complete each equation.

a. $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{5}{4}$

b. $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=\frac{4}{3}$

c. $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{7}{6}$

d. $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\frac{5}{2}$

Ask, *What happens to the numerator and denominator when the fraction is exactly one whole?* (They are the same.) *What are these types of fractions called?* (Improper fractions.) *What equation can we write to show the total number of half pizzas?* (**SMP6**) Write the equation $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\frac{6}{2}$. Then ask, *What different combinations of halves make six-halves?* (**SMP7**) Record each combination on the board and make sure students explain how they figured it out.

Write the word problem, as shown, on the board, and have the students record the skip counting by $\frac{1}{4}$ on a number line to solve the problem (**SMP4**). Then have them write an equation to show the total amount. Allow time for them to compare their number lines and equations with another student. Invite volunteers to share their solution. Emphasize the difference between the numerator and denominator.

One batch of apple oatmeal cookies uses $\frac{1}{4}$ cup of sugar. How much sugar is needed for 5 batches?

Work through the Step In discussion (Student Journal 8.7) with the whole class. Point to $\frac{4}{3}$ on the number line and ask, *What does the 3 tell you?* (The number of jumps needed to reach the whole.) *What does the 4 tell you?* (The number of jumps completed.) Highlight how the fourth jump gets them past the whole (one). Read the Step Up and Step Ahead instructions with the students. Make sure they know what to do, then have them work independently to complete the tasks.

Step 4 Reflecting on the work

Discuss the students' answers to Student Journal 8.7. Refer to Question 3 and ask, *How did you decide which fractions were greater than 1 but less than 2?* Encourage students to draw diagrams on the board to illustrate their explanations. (**SMP4**)

Applications

If time allows, have the students complete this Investigation and/or Problem solving activity.

Investigation: Working with improper fractions

Each small group of students will need:

- access to paper and pencils

Write the investigation question, as shown, on the board and read it with the students. Organize students into small groups and have them work together to investigate and record as many different combinations as possible. Observe the methods they use, such as working with a number line, drawing shapes, or simply recording the equations. Afterward, invite a volunteer from each group to share one combination at a time until all the recorded combinations have been shared. Then encourage groups to describe the steps they followed. Highlight efficient and systematic methods. Afterward, lead a class discussion about whether there are any other possible combinations.

Investigation question

How many different combinations of fractions will add to a total of $\frac{13}{6}$?

Problem solving: Skip counting by unit fractions

Write the following problem on the board, then read it with the students:

What number am I?
I am greater than one whole but less than 2.
On a number line you can skip count by $\frac{1}{8}$ to reach me.
You can use an area model to represent me.
My numerator is the product of a sixes multiplication fact.

Organize the students into pairs to work together to solve the problem. Encourage them to record the equations they used. Afterward, bring the class back together and have them share their thinking with the class and the solution ($\frac{12}{8}$).

Small group differentiation

Extra help

Open the *Flare Fractions* (*Length*) online tool and display a number line representing fourths, as shown. Have a student confirm that the number line represents fourths. Guide the students to count by fourths as you draw the matching number of jumps above the number line. Repeat with other fractions.

Extra practice

Each group of students will need:

- 1 number cube labeled: 2, 2, 3, 3, 4, 4

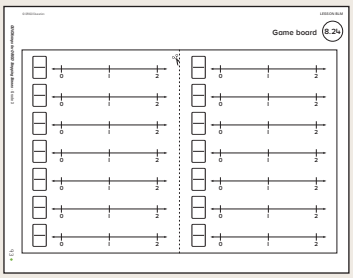
Each student will need:

- 1 game board from Blackline Master 8.24

Organize students into small groups and distribute the resources. Explain that one student rolls the cube and decides whether the result will be a numerator or a denominator. The result is written in the answer box beside the first number line. The student rolls the cube again and writes the result to complete the fraction. The student splits the number line beside their fraction to match the denominator, then draws jumps to match the numerator. The distance between each whole number represents one whole. The other students take turns to repeat the activity. Play continues for 7 rounds. Points are given according to the size of the jump in each round. 1 point is given for fractions less than 1, 2 points are given for fractions equal to 1, and 3 points are given for fractions greater than 1. The student with the greatest number of points wins.

For example, if a roll of 2 is chosen as the numerator and a roll of 3 as denominator then the fraction $\frac{2}{3}$ should be written and the distance between each whole number on the number line split into thirds with two jumps of $\frac{1}{3}$ shown above. This would be awarded 1 point.

Blackline Master 8.24



8.7 Common fractions: Identifying improper fractions on a number line

Step In

One batch of 12 muffins needs $\frac{2}{3}$ cup of mashed banana.

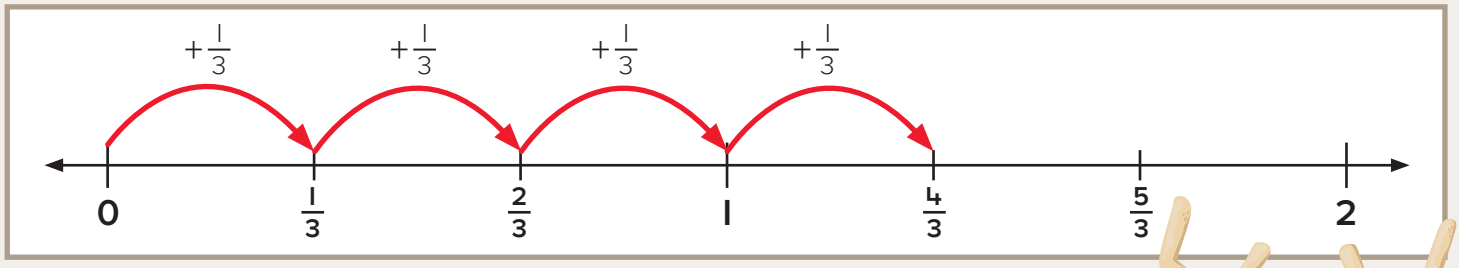
Maka wants to make 2 batches but he only has a $\frac{1}{3}$ measuring cup.

What can he do to measure the correct amount of banana for 2 batches of muffins?

Maka can use the $\frac{1}{3}$ measuring cup two times for one batch, so he can use it four times for two batches.

How could you show your thinking on a number line?

What fraction could you write to show the total amount of banana?

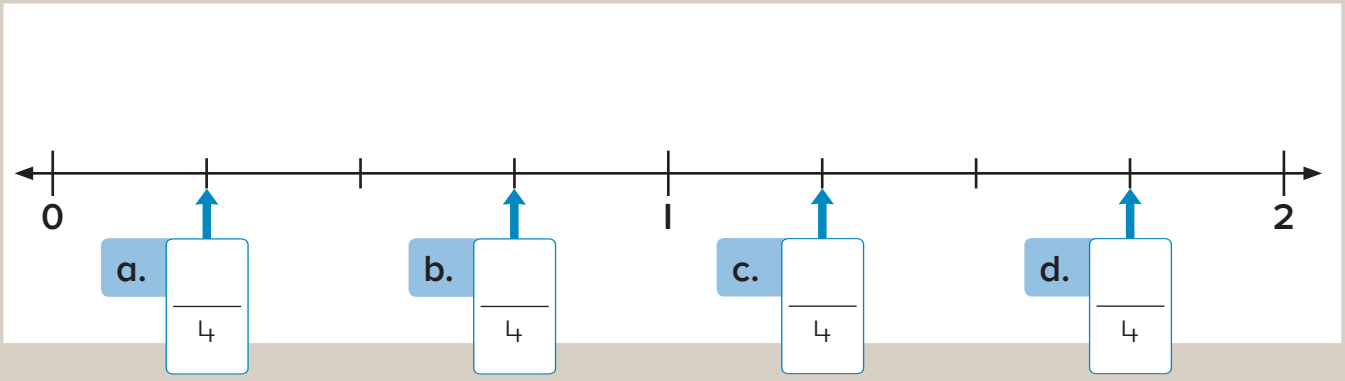


What do you notice about the fraction $\frac{4}{3}$?

The numerator is greater than the denominator. I can see on the number line that $\frac{4}{3}$ is greater than 1.

Step Up

1. On this number line, the distance from 0 to 1 is one whole. Write the fraction that should be in each box. Draw jumps to help you.



2. On each number line, the distance from 0 to 1 is one whole.
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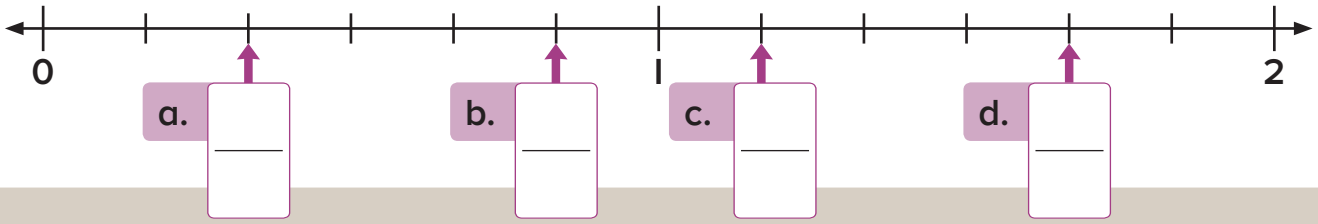
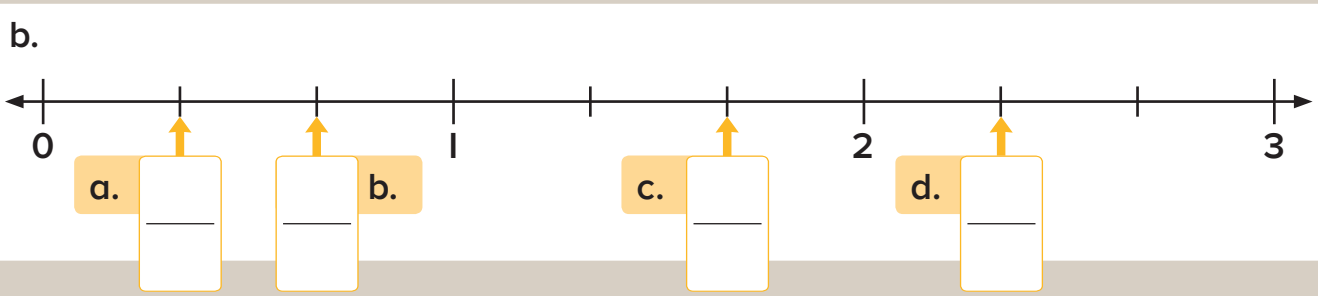
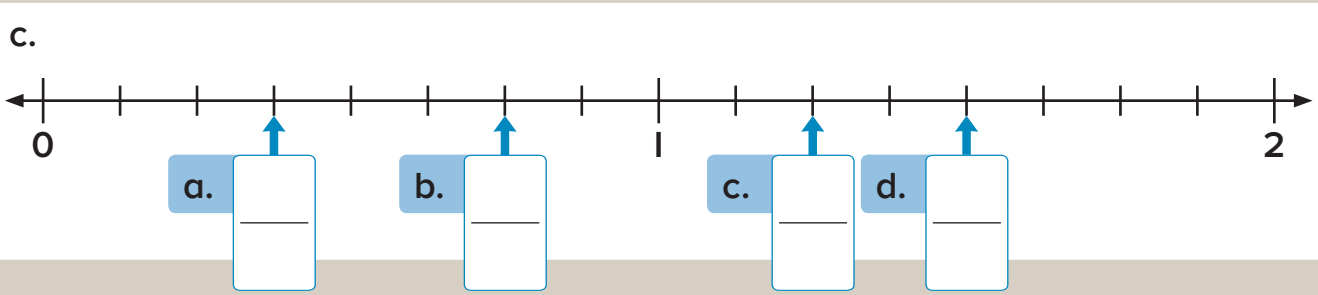
a.



b.



c.



3. Use the fractions you wrote on the number lines above.

a. List the fractions that are less than 1.

b. List the fractions that are greater than 1 but less than 2.

Step Ahead

Complete each equation.

a. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} =$ ____

b. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} =$ ____

c. $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} =$ ____

d. $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} =$ ____

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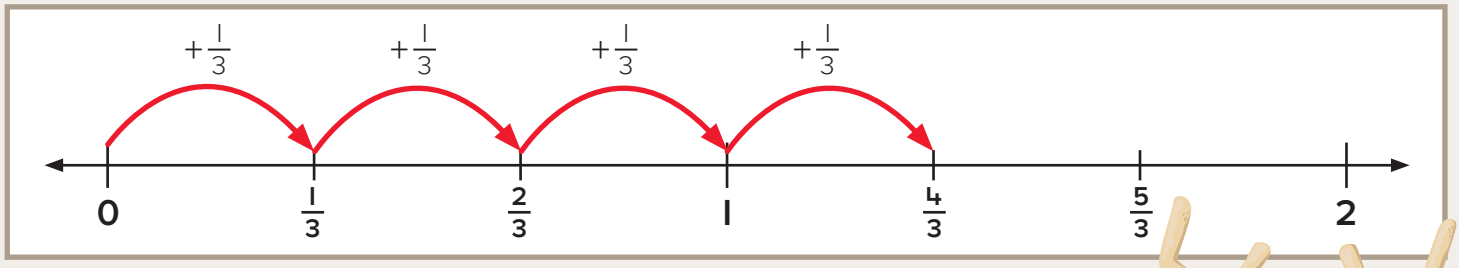

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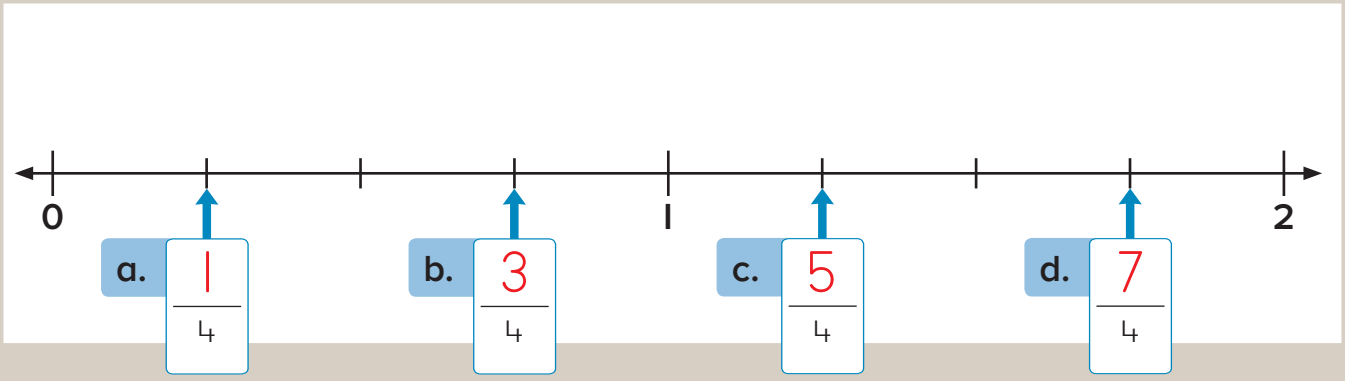
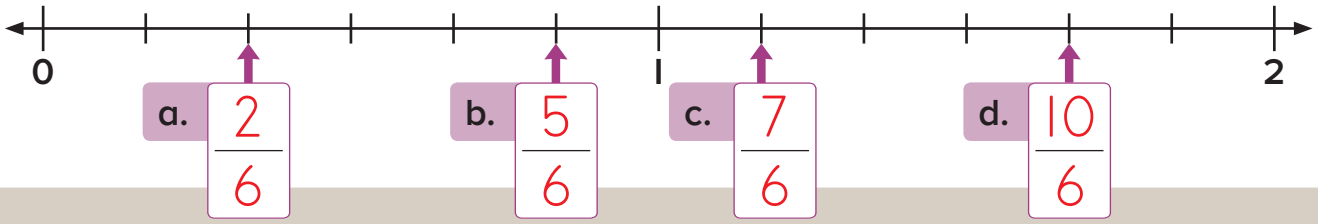
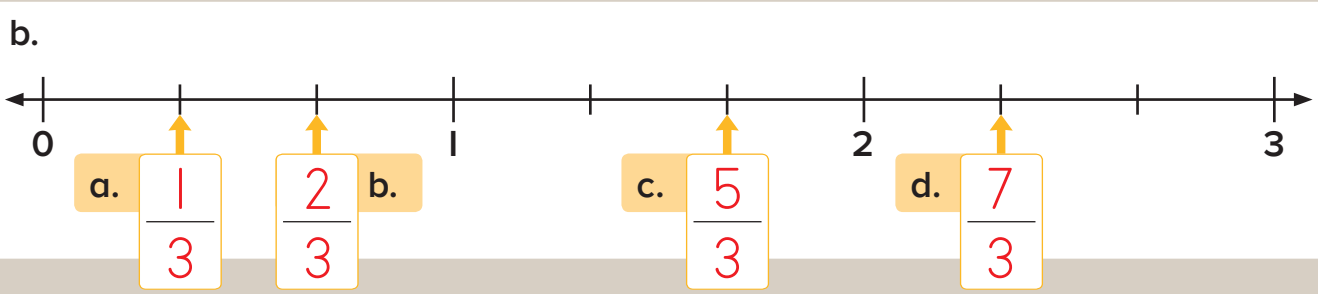


2. On each number line, the distance from 0 to 1 is one whole.
Write the fraction that should be in each box. Draw jumps to help you.

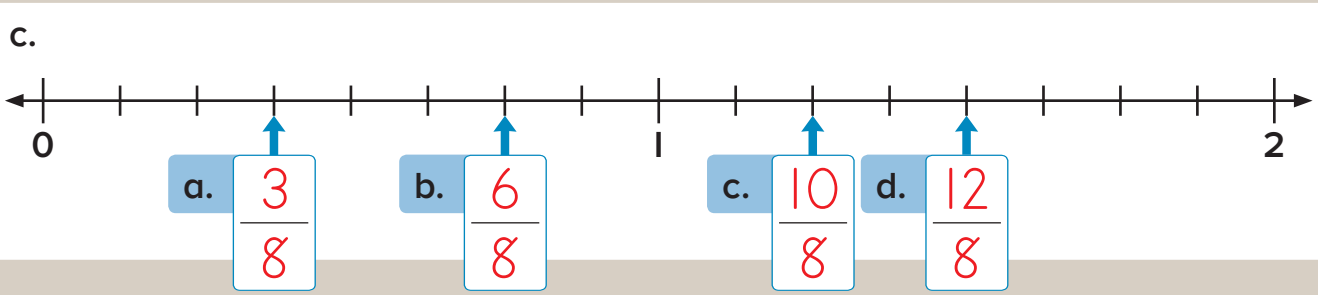
a.



b.



c.



3. Use the fractions you wrote on the number lines above.

a. List the fractions that are less than 1.

$\frac{2}{6}$ $\frac{5}{6}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{8}$ $\frac{6}{8}$

b. List the fractions that are greater than 1 but less than 2.

$\frac{7}{6}$ $\frac{10}{6}$ $\frac{5}{3}$ $\frac{10}{8}$ $\frac{12}{8}$

Step Ahead

Complete each equation.

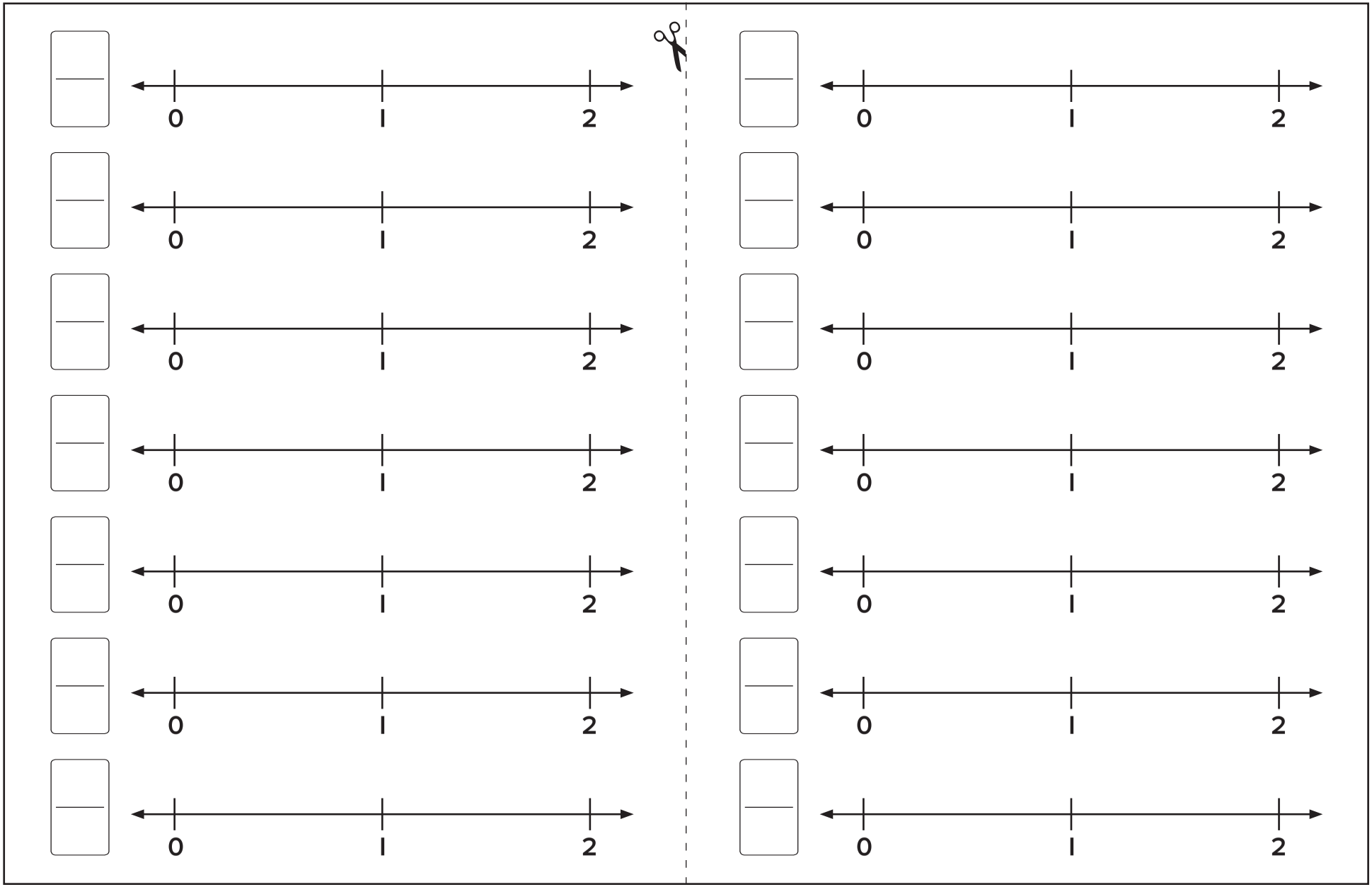
a. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{5}{4}$

b. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{4}{3}$

c. $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{7}{6}$

d. $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{5}{2}$

Game board 8.24



8

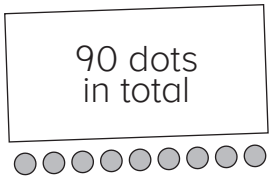
Answers will vary.

1. Write a division fact to match each picture.

a.

18 ÷ 2 = 9

b.



90 ÷ 9 = 10

2. Solve the problem. Show your thinking.

Mom spends $48 on tickets to the school dance. Tickets cost $6 each. How many tickets does she buy?

8 tickets

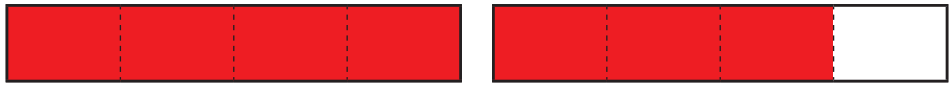
3. Write multiplication facts that will help you complete the division facts.

a.	9 × 2 = 18	so	18 ÷ 9 = 2
b.	9 × 6 = 54	so	54 ÷ 9 = 6
c.	7 × 7 = 49	so	49 ÷ 7 = 7
d.	6 × 6 = 36	so	36 ÷ 6 = 6

4. Each strip is one whole. Color parts to show each fraction.

a. four-fourths

b. seven-fourths

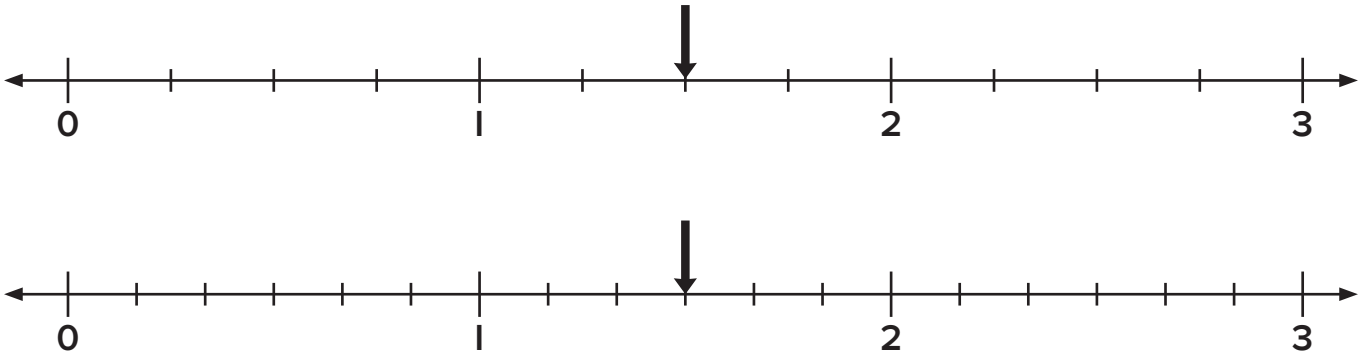


Q1 3.OA.A.2 Q2 3.OA.A.3
Q3 3.OA.A.4 + 3.OA.B.6 + 3.OA.C.7 Q4 3.NF.A.1

8

✻ Answers will vary.

5. The distance from 0 to 1 on each number line is one whole.



a. Write the two equivalent fractions that are shown by the arrows.

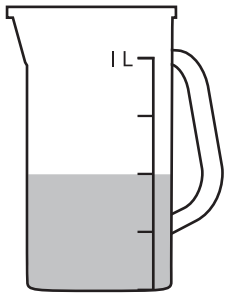
$\frac{6}{4}$ $\frac{9}{6}$

b. Use the number lines to write two fractions that are equal to 1.

$\frac{4}{4}$ $\frac{6}{6}$

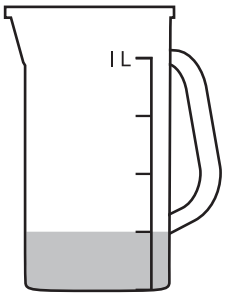
6. Write the amount of water in each pitcher.

a.



$\frac{1}{2}$ L

b.



$\frac{1}{4}$ L

7. Solve each problem. Show your thinking.

a. Four cans weigh a total of 1,000 g. How much does one can weigh?

✻

250 g

b. A pitcher holds two liters of water. The water from the pitcher is poured equally into 8 small glasses. What fraction of a liter is in each glass?

✻

$\frac{1}{4}$ L

Q5 3.NF.A.2 + 3.NF.A.2b + 3.NF.A.3 + 3.NF.A.3a + 3.NF.A.3b + 3.NF.A.3c

Q6+7 3.MD.A.2 Q7 3.OA.D.8

8.7 Fracciones comunes: Identificando fracciones impropias en una recta numérica

Conoce

Se necesitan $\frac{2}{3}$ de taza de puré de banana para hacer una hornada de 12 *muffins*.

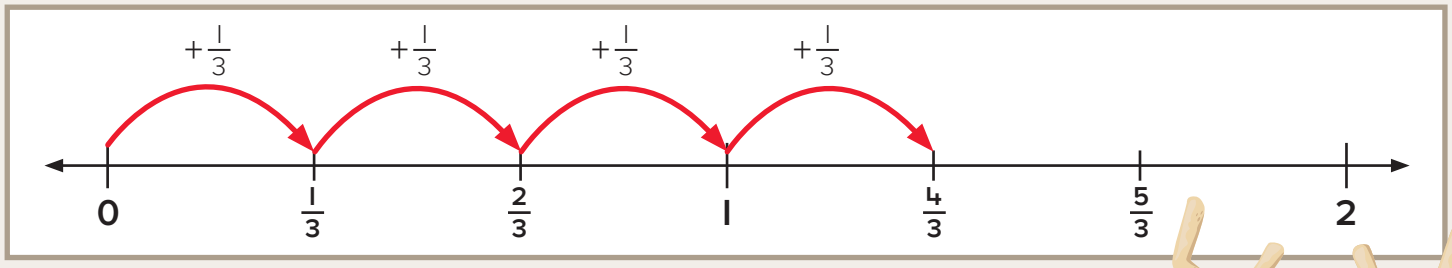
Maka quiere hacer 2 hornadas pero solo tiene una taza de medir de $\frac{1}{3}$.

¿Qué puede hacer él para medir la cantidad correcta de banana para hacer 2 hornadas de *muffins*?

Maka puede utilizar la taza de medir de $\frac{1}{3}$ dos veces para una hornada, por lo tanto puede utilizarla cuatro veces para dos hornadas.

¿Cómo podrías indicar tu razonamiento en una recta numérica?

¿Qué fracción podrías escribir para indicar la cantidad total de banana?

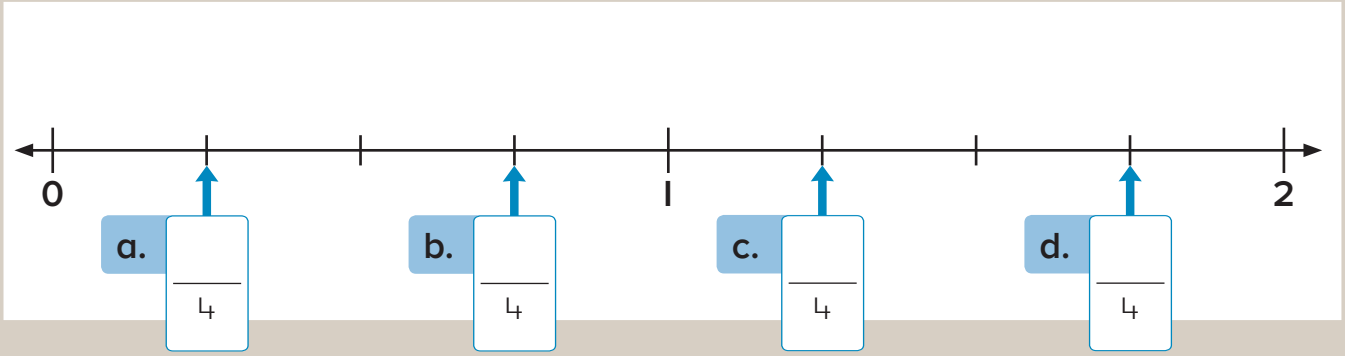


¿Qué notas en la fracción $\frac{4}{3}$?

El numerador es mayor que el denominador. Puedo ver en la recta numérica que $\frac{4}{3}$ es mayor que 1.

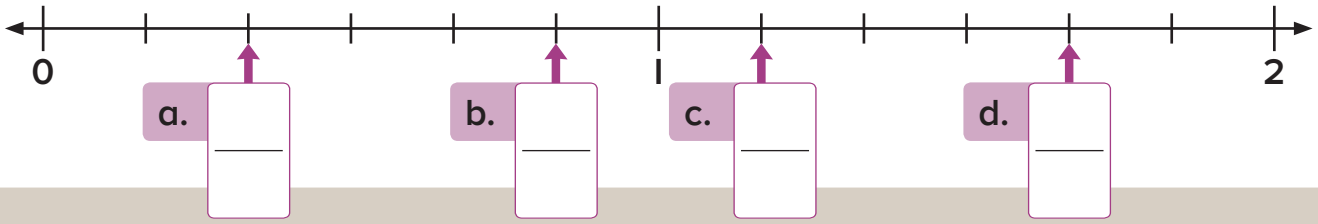
Intensifica

1. En esta recta numérica la distancia de 0 a 1 es un entero. Escribe la fracción que debería estar en cada casilla. Dibuja saltos como ayuda.

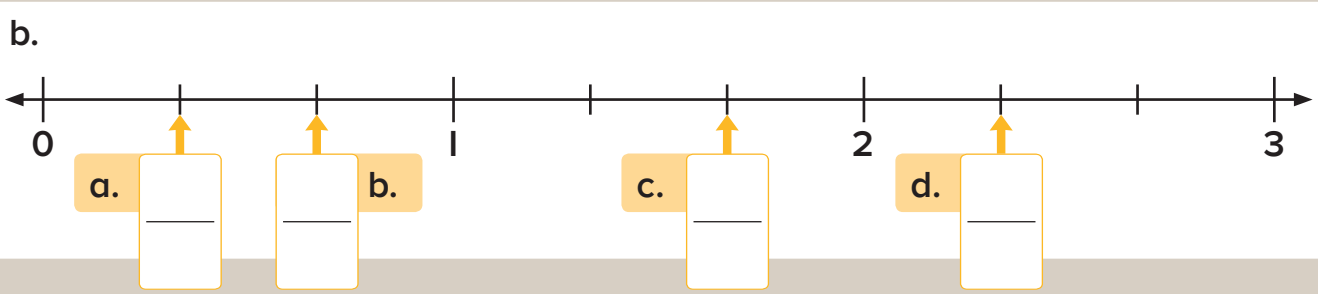


2. En esta recta numérica la distancia de 0 a 1 es un entero. Escribe la fracción que debería estar en cada casilla. Dibuja saltos como ayuda.

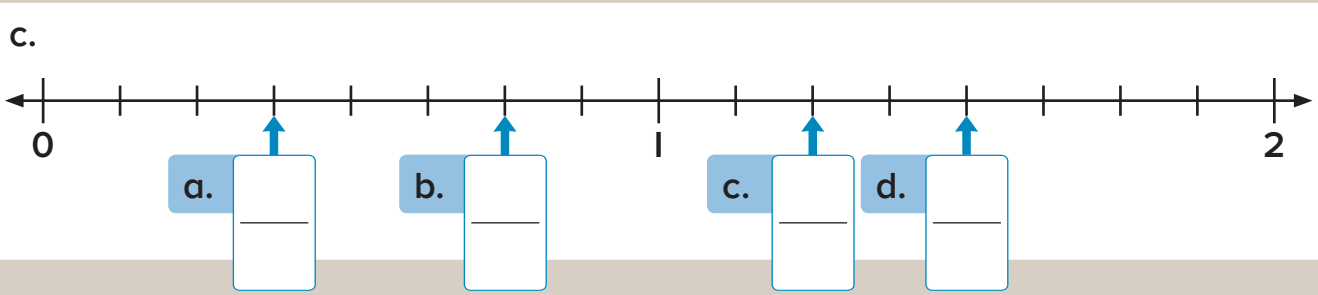
a.



b.



c.



3. Utiliza las fracciones que escribiste en las rectas numéricas de arriba.

a. Haz una lista de las fracciones menores que 1.

b. Haz una lista de las fracciones mayores que 1 pero menores que 2.

Avanza

Completa cada ecuación.

a. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} =$ ____

b. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} =$ ____

c. $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} =$ ____

d. $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} =$ ____

8.7 Fracciones comunes: Identificando fracciones impropias en una recta numérica

Conoce

Se necesitan $\frac{2}{3}$ de taza de puré de banana para hacer una hornada de 12 *muffins*.

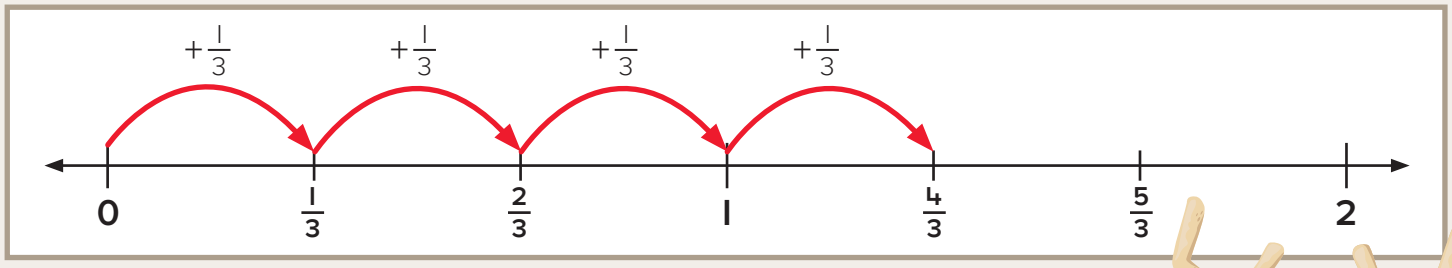
Maka quiere hacer 2 hornadas pero solo tiene una taza de medir de $\frac{1}{3}$.

¿Qué puede hacer él para medir la cantidad correcta de banana para hacer 2 hornadas de *muffins*?

Maka puede utilizar la taza de medir de $\frac{1}{3}$ dos veces para una hornada, por lo tanto puede utilizarla cuatro veces para dos hornadas.

¿Cómo podrías indicar tu razonamiento en una recta numérica?

¿Qué fracción podrías escribir para indicar la cantidad total de banana?

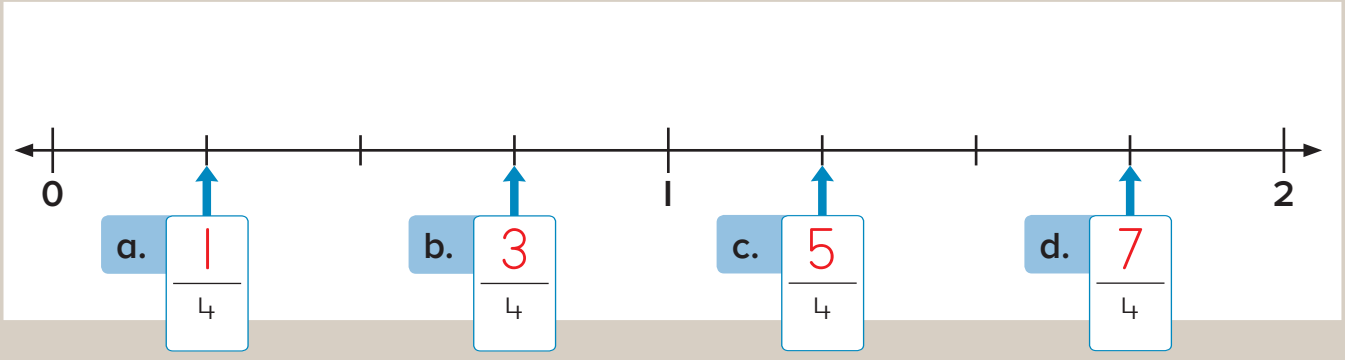


¿Qué notas en la fracción $\frac{4}{3}$?

El numerador es mayor que el denominador. Puedo ver en la recta numérica que $\frac{4}{3}$ es mayor que 1.

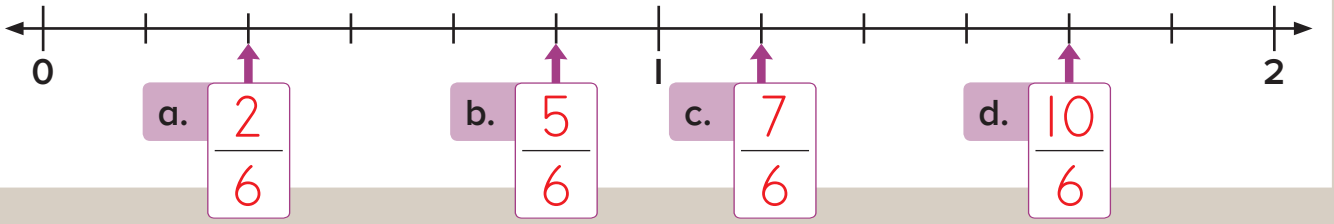
Intensifica

1. En esta recta numérica la distancia de 0 a 1 es un entero. Escribe la fracción que debería estar en cada casilla. Dibuja saltos como ayuda.

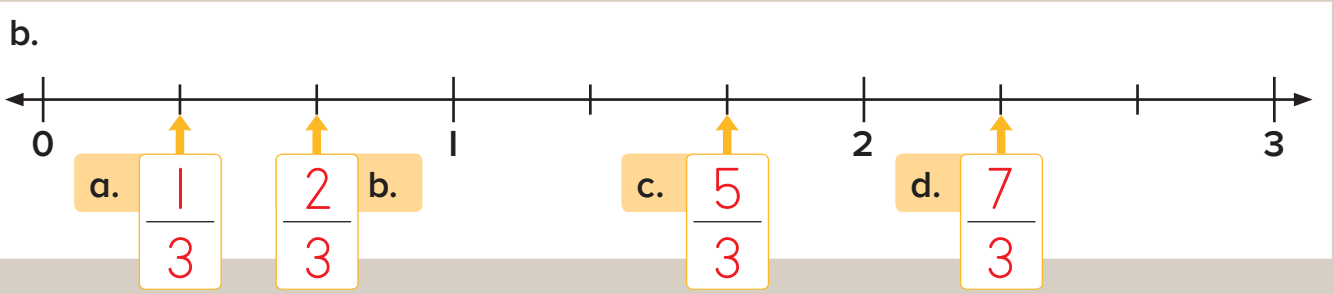


2. En esta recta numérica la distancia de 0 a 1 es un entero. Escribe la fracción que debería estar en cada casilla. Dibuja saltos como ayuda.

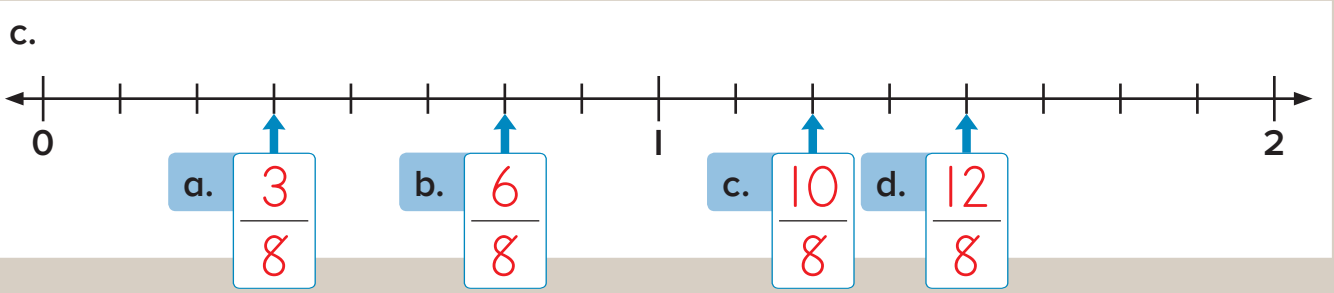
a.



b.



c.



3. Utiliza las fracciones que escribiste en las rectas numéricas de arriba.

a. Haz una lista de las fracciones menores que 1.

$\frac{2}{6}$ $\frac{5}{6}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{8}$ $\frac{6}{8}$

b. Haz una lista de las fracciones mayores que 1 pero menores que 2.

$\frac{7}{6}$ $\frac{10}{6}$ $\frac{5}{3}$ $\frac{10}{8}$ $\frac{12}{8}$

Avanza

Completa cada ecuación.

a. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{5}{4}$

b. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{4}{3}$

c. $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{7}{6}$

d. $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{5}{2}$