

Unpacking Mathematical Operations – Addition & Subtraction

CAMT 2019
San Antonio

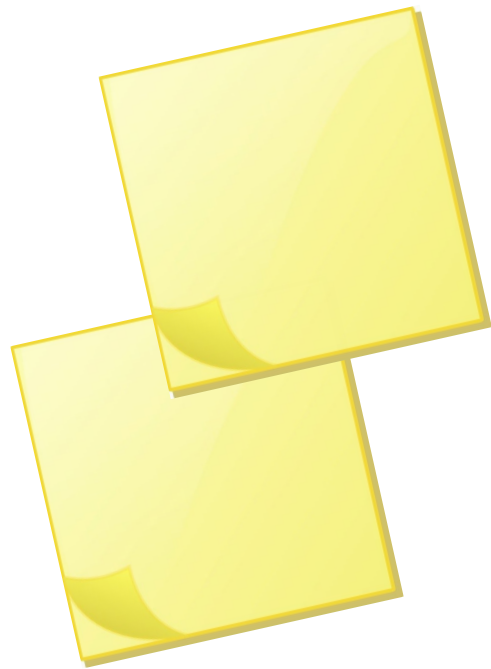
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(s_moore@origomath.com)

Take 2 sticky notes and write one addition or subtraction word problem on each:

- 1-step problems only
- Use 1- or 2-digit numbers
- Draw a picture to represent your solution

Put them aside for now.



Who's Here Today?

Raise your hand if you are:


- A primary grades teacher
- An intermediate grades teacher
- A middle school teacher
- A high school teacher
- A math coach or specialist
- An administrator
- A teacher educator
- Other??



47% of word problem errors occurred before any sort of calculation process took place.

(Newman, 1977, as cited in Watson, 1980)

Watson, I. (1980). Investigating errors of beginning mathematicians. *Educational Studies in Mathematics*, 11(3), p.319–329. <https://doi.org/10.1007/BF00697743>



How are these problems the same? Different?

Draw a picture for each.

Problem 1

There are 26 students in Mrs. Amadi's class. 15 left to go to PE. How many students remain in the classroom?

Problem 2

There are 26 students in Mrs. Amadi's class. 15 are doing work on the computer. The rest are reading at their seats. How many are reading?



Match each problem to the representations that best match it.
Explain your reasoning.

Problem 1

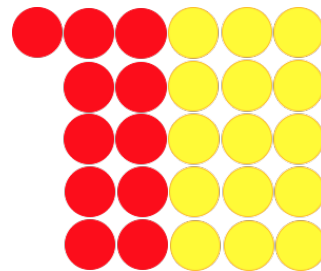
There are 26 students in Mrs. Amadi's class. 15 left to go to PE. How many students remain in the classroom?

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There are 26 students in Mrs. Amadi's class. 15 are doing work on the computer. The rest are reading at their seats. How many are reading?

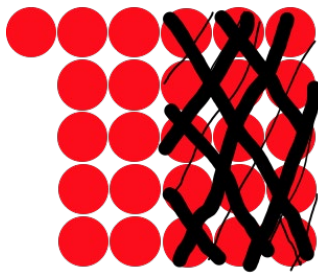
A

Computer	Reading
Students	



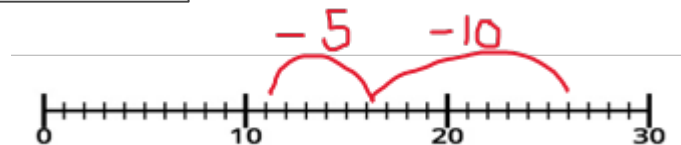
B

C



$$\begin{array}{r} 26 \\ - 15 \\ \hline 11 \end{array}$$

E

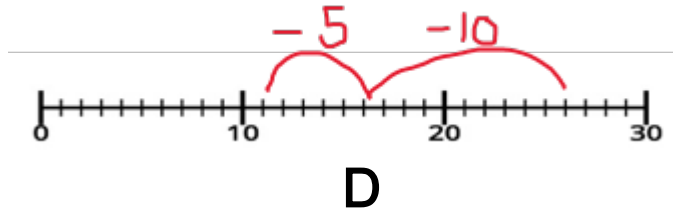
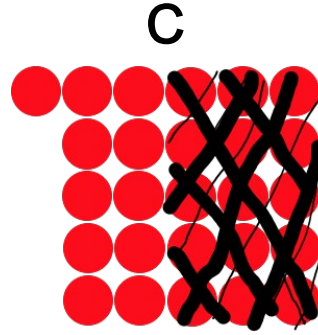


D

Active Problem Situation

Problem 1

There are 26 students in Mrs. Amadi's class. 15 left to go to PE. How many students remain in the classroom?



$$\begin{array}{r} 26 \\ - 15 \\ \hline 11 \end{array} \quad \text{E}$$

Separate Problem Type (Result Unknown)

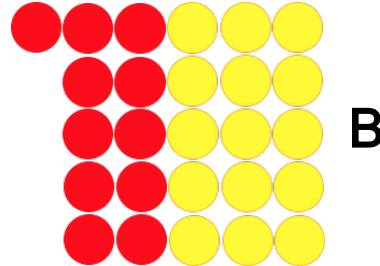
Non-Active Problem Type

Problem 2

There are 26 students in Mrs. Amadi's class. 15 are doing work on the computer. The rest are reading at their seats. How many are reading?

A

Computer	Reading
Students	



$$\begin{array}{r} 26 \\ - 15 \\ \hline 11 \end{array} \quad \text{E}$$

Part-Part-Whole Problem Type (Part Unknown)



Let's look at the TEKS

Grade K

(3) Number and operations. The student applies mathematical process standards to develop an understanding of addition and subtraction situations in order to solve problems. The student is expected to:

- (A) model the action of **joining** to represent addition and the action of **separating** to represent subtraction;
- (B) solve word problems using objects and drawings to find sums up to 10 and differences within 10; and
- (C) explain the strategies used to solve problems involving adding and subtracting within 10 using spoken words, concrete and pictorial models, and number sentences.

Grade 1

(3) Number and operations. The student applies mathematical process standards to develop and use strategies for whole number addition and subtraction computations in order to solve problems. The student is expected to:

- (A) use concrete and pictorial models to determine the sum of a multiple of 10 and a one-digit number in problems up to 99;
- (B) use objects and pictorial models to solve word problems involving **joining, separating, and comparing sets** within 20 and **unknowns as any one of the terms** in the problem such as $2 + 4 = []$; $3 + [] = 7$; and $5 = [] - 3$;
- (C) compose 10 with two or more addends with and without concrete objects;
- (D) apply basic fact strategies to add and subtract within 20, including making 10 and decomposing a number leading to a 10;
- (E) explain strategies used to solve addition and subtraction problems up to 20 using spoken words, objects, pictorial models, and number sentences; and
- (F) generate and solve problem situations when given a number sentence involving addition or subtraction of numbers within 20.

Grade 2

Students **identify situations in which addition and subtraction are useful to solve problems**. Students develop a variety of strategies to use efficient, accurate, and generalizable methods to add and subtract multi-digit whole numbers.

(6) Number and operations. The student applies mathematical process standards to connect repeated addition and subtraction to multiplication and division situations that involve equal groupings and shares. The student is expected to:

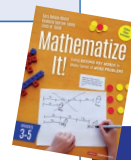
- (A) model, create, and describe contextual multiplication situations in which **equivalent sets of concrete objects are joined**; and
- (B) model, create, and describe contextual division situations in which a **set of concrete objects is separated into equivalent sets**.

The Work of Addition & Subtraction

- *Are these problems about action or relationships?*
- *If you see action, what is the change?*
- *If you see relationships, how would you describe the relationships? Are they about groups or about comparing?*

FIGURE 1.2 ADDITION AND SUBTRACTION PROBLEM SITUATIONS

ACTIVE SITUATIONS				
	Result Unknown	Change Addend Unknown	Start Addend Unknown	
Add-To	<p>Paulo counted out 75 crayons and put them in the basket. Then he found 23 more crayons under the table. He added them to the basket. How many crayons are now in the basket?</p> $75 + 23 = x$ $23 = x - 75$	<p>Paulo counted out 75 crayons and put them in the basket. Then he found some more crayons under the table. He added them to the basket and now there are 98 crayons in the basket. How many crayons were under the table?</p> $75 + x = 98$ $75 = 98 - x$	<p>Paulo was organizing the crayons at his table. He found 23 crayons under the table and added them to the basket. When he counted, there are now 98 crayons in the basket. How many crayons were in the basket before Paulo looked under the table for crayons?</p> $x + 23 = 98$ $98 - 23 = x$	
Take-From	<p>There are 26 students in Mrs. Amadi's class. 15 left to get ready to play in the band at the assembly. How many students are not in the band?</p> $26 - 15 = x$ $15 + x = 26$	<p>There are 26 students in Mrs. Amadi's class. After the band students left the class for the assembly, there were 11 students still in the classroom. How many students are in the band?</p> $26 - x = 11$ $x + 11 = 26$	<p>15 band students left Mrs. Amadi's class to get ready to play in the assembly. There were 11 students left in the classroom. How many students are in Mrs. Amadi's class?</p> $x - 15 = 11$ $15 + 11 = x$	
RELATIONSHIP (NON-ACTIVE) SITUATIONS				
	Total Unknown	One Part Unknown	Both Parts Unknown	
Part-Part-Whole	<p>The 4th grade held a vote to decide where to go for the annual field trip. 32 students voted to go to the ice skating rink. 63 voted to go to the local park. How many students are in the 4th grade?</p> $32 + 63 = x$ $x - 63 = 32$	<p>The 4th grade held a vote to decide where the 95 students should go for their annual field trip. 32 students voted to go to the ice skating rink. The rest chose the local park. How many voted to go to the park?</p> $32 + x = 95$ $x = 95 - 32$	<p>The 4th grade held a vote to decide where the 95 students should go for their annual field trip. Some students voted to go to the ice skating rink and others voted to go to the local park. What are some possible combinations of votes?</p> $x + y = 95$ $95 - x = y$	
	Difference Unknown	Greater Quantity Unknown	Lesser Quantity Unknown	
Additive Comparison	<p>Jessie and Jo both collect baseball cards. Jo has 71 cards and Jessie has 53 cards. How many fewer cards does Jessie have than Jo?</p> $53 + x = 71$ $53 = 71 - x$	<p>Jessie and Jo both collect baseball cards. Jessie has 53 cards and Jo has 18 more cards than Jessie has. How many baseball cards does Jo have?</p> $53 + 18 = x$ $x - 18 = 53$	<p>Jessie and Jo both collect baseball cards. Jo has 71 cards and Jessie has 18 fewer cards than Jo. How many baseball cards does Jessie have?</p> $71 - 18 = x$ $x + 18 = 71$	



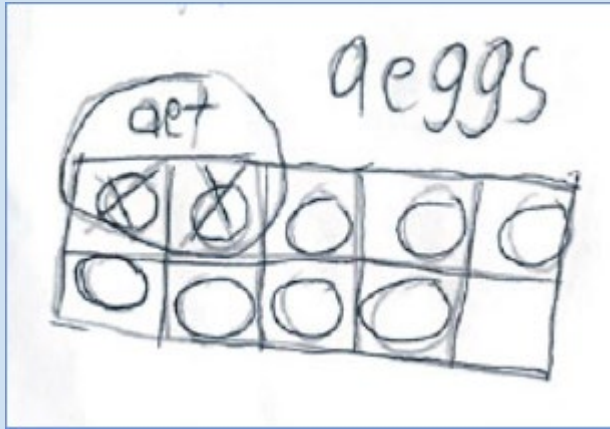
Describe what is happening.

Show a solution in words, pictures, and numbers .

There were 9 eggs in the carton. Emily ate two eggs for her breakfast. How many eggs are left in the carton?



There were 9 eggs in the carton. Emily ate two eggs for her breakfast. How many eggs are left in the carton?



A

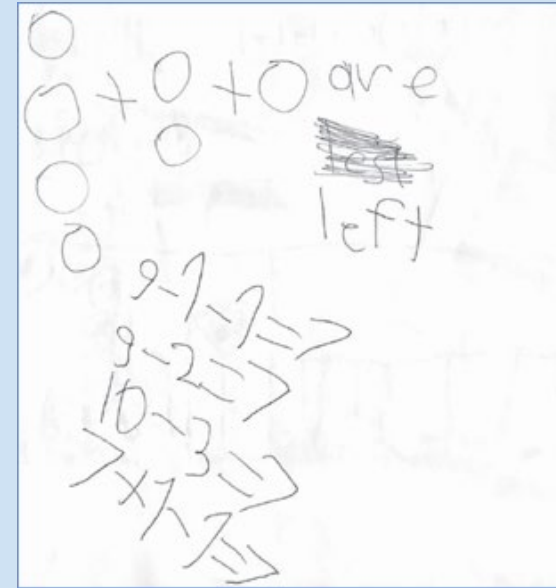
$$9 - 2 = 7$$

|||||

B

$$9 - 2 = 7$$

C



D

Where is the unknown?

ACTIVE SITUATIONS				
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Where is the unknown?

There were 9 eggs in the carton. Emily ate two eggs for her breakfast. How many eggs are left in the carton?

What is the starting quantity? 9 eggs

What is the change? 2 eggs

What is the resulting quantity? x eggs

Rewrite the problem with a different term unknown.

Context or Computation?

Do the students' representations (particularly drawings), reflect the context of the problem or their preferred computation strategy?

What would a context-focused representation of this problem look like?



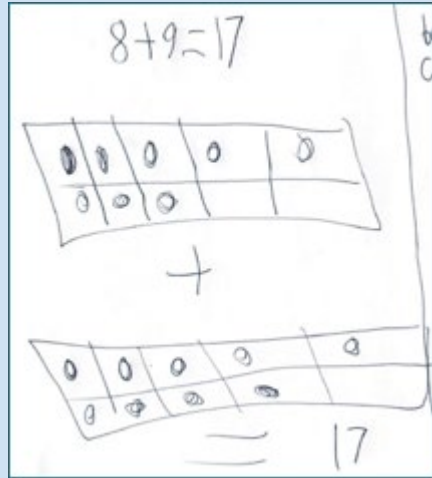
Describe what is happening.

Show a solution in words, pictures, and numbers .

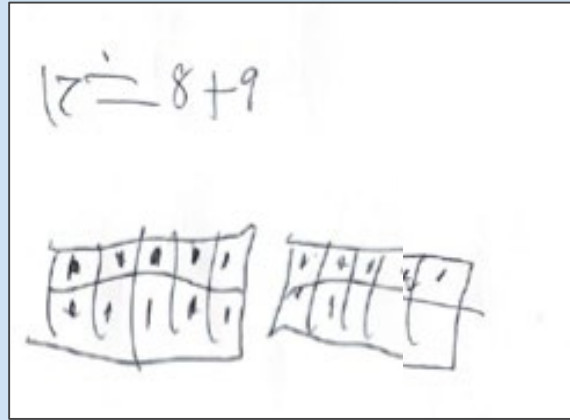
The class voted for their favorite flavor of ice cream. 8 students voted for vanilla and 9 students voted for chocolate. How many students are in the class?



The class voted for their favorite flavor of ice cream. 8 students voted for vanilla and 9 students voted for chocolate. How many students are in the class?



A



B

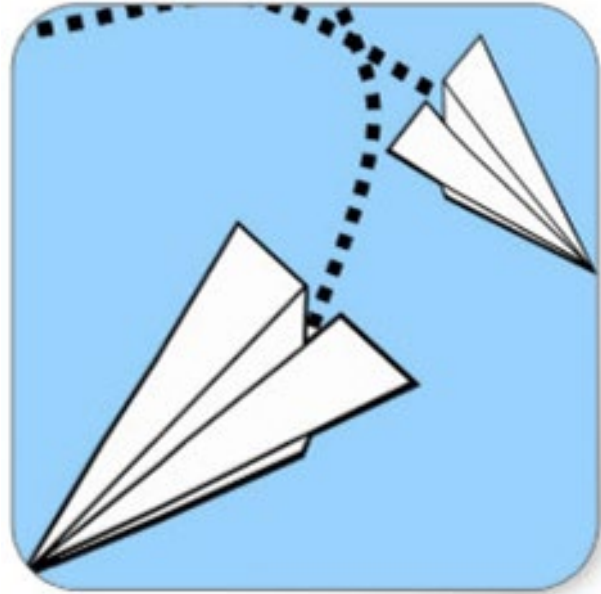


C

Describe what is happening.

Show a solution in words, pictures, and numbers .

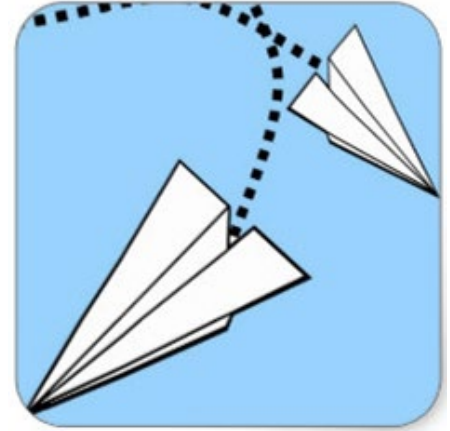
Jessie's paper airplane flew
14 feet. Jessie's paper
airplane flew 5 feet more
than Jo's paper airplane.
How far did Jo's paper
airplane fly?



Examining Your Work

Turn and share your solution with your neighbor.

- Tell the story of the problem.
- How does your drawing represent the problem?
- What incorrect responses would you expect from students?
- What type of problem is this?



Big Books

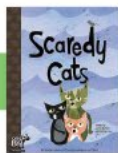
Grade K



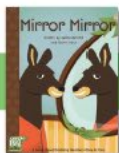
Sweet Dreams
Counting
Quantities to Ten



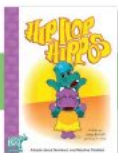
Ten Happy Hens
Subtraction
(Take From)



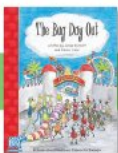
Scaredy Cats
Combinations
to Ten



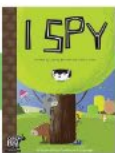
Mirror Mirror
Doubling Numbers
One to Five



Hip Hop Hippos
Numbers and
Relative Position



The Bug Day Out
Numbers Eleven
to Sixteen



I Spy
Positional
Language



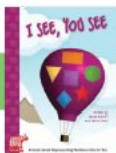
**The Clown's New
Clothes**
Length



Paint a Rainbow
Ordinal Numbers



**Mice, Mice
Everywhere**
Addition
(Put Together)

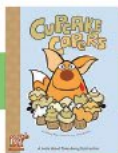


I See, You See
Identifying
Quantities
One to Ten



Perfect Patterns
Making and
Describing Patterns

Grade 1



Cupcake Capers
Subtraction
(Take From)



Stella's Store
Skip Counting
by Five



A Simple Stew
Growing
Number Patterns



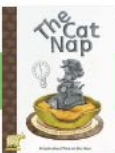
The Best Bug
Non-Standard
Units of Length



The Flower Pot Plan
Representing Data



The Space Party
Multiplication
(Equal Groups)



The Cat Nap
Time on the Hour



Muddy, Muddy Mess
3D Objects and
2D Shapes



Addtron
Using Doubles
to Add



How Many Legs?
Number
Combinations



Shoes in Twos
Relating Counting
to Addition

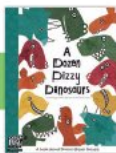


**Patterns Here,
Patterns There!**
Patterns

Grade 2



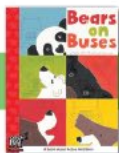
The Fun Machine
Addition and
Subtraction



**A Dozen Dizzy
Dinosaurs**
Division
(Equal Groups)



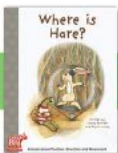
Munch and Crunch
Doubling
and Halving



Bears on Buses
Addition
(Add To)



The Pirate's Gold
Division
(Sharing)



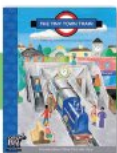
Where is Here?
Position, Direction,
and Movement



Joe's Carrots
Subtraction
(Unknown Addend)



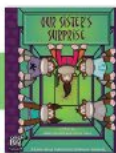
The Big Bug Band
Multiplication
(Arrays)



**The Tiny
Town Train**
Time Past
the Hour



Clowning Around
Collecting and
Representing Data



**Our Sister's
Surprise**
Subtraction
(Unknown Addend)



A Bear's Share
Division
(Sharing)

Revisit Post it Notes

Look again at the problems you put on the post-it notes.

What problem type(s) did you represent?

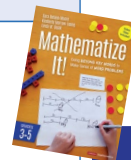


The Work of Addition & Subtraction

- *Are these problems about action or relationships?*
- *If you see action, what is the change?*
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FIGURE 1.2 ADDITION AND SUBTRACTION PROBLEM SITUATIONS

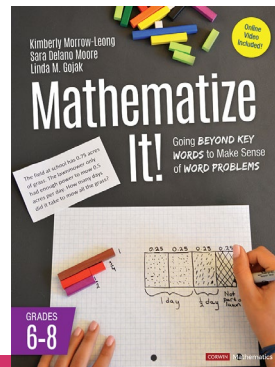
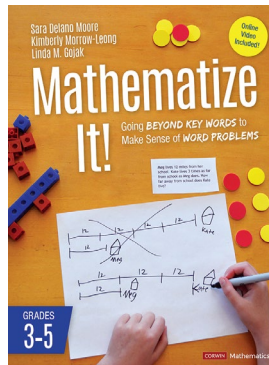
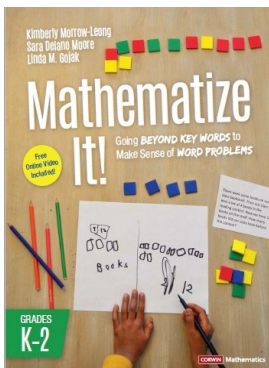
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RELATIONSHIP (NON-ACTIVE) SITUATIONS				
	Total Unknown	One Part Unknown	Both Parts Unknown	
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	Difference Unknown	Greater Quantity Unknown	Lesser Quantity Unknown	
Additive Comparison	<p>Jessie and Jo both collect baseball cards. Jo has 71 cards and Jessie has 53 cards. How many fewer cards does Jessie have than Jo?</p> $53 + x = 71$ $53 = 71 - x$	<p>Jessie and Jo both collect baseball cards. Jessie has 53 cards and Jo has 18 more cards than Jessie has. How many baseball cards does Jo have?</p> $53 + 18 = x$ $x - 18 = 53$	<p>Jessie and Jo both collect baseball cards. Jo has 71 cards and Jessie has 18 fewer cards than Jo. How many baseball cards does Jessie have?</p> $71 - 18 = x$ $x + 18 = 71$	



Thank you!

Slides available in the conference app or at
<https://www.origoeducation.com/camt19>

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