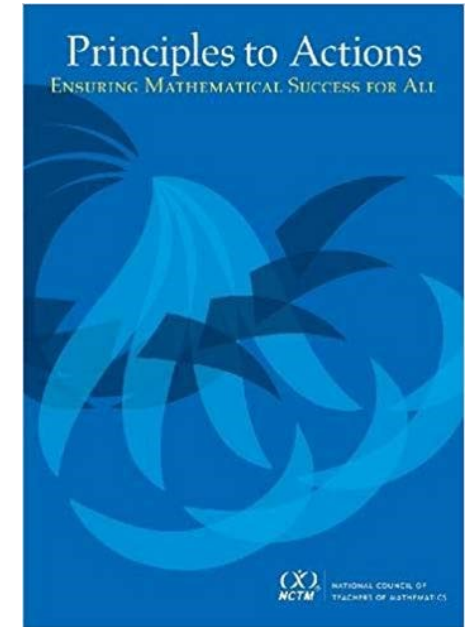
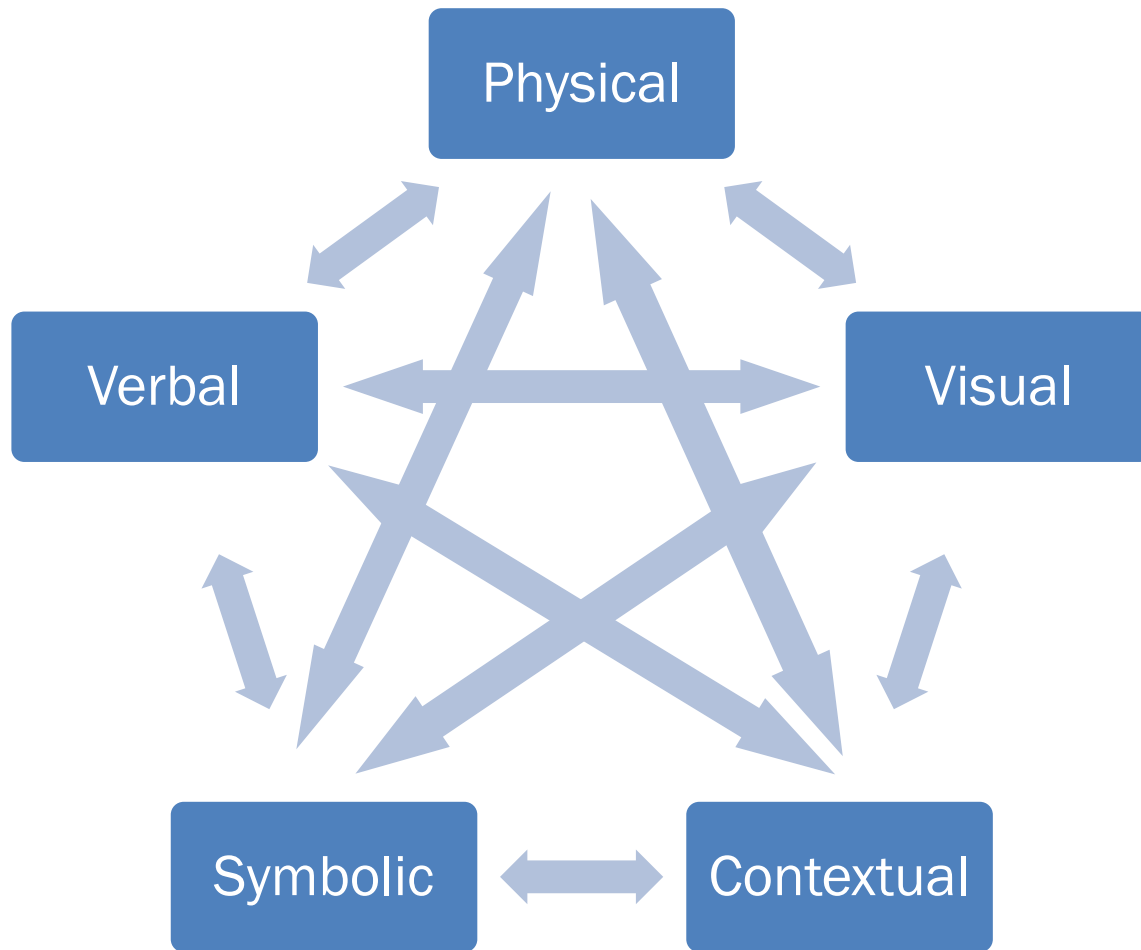


# Using Multiple Representations for Deep Mathematics Learning

CAMT 2019  
San Antonio

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# Use and connect multiple representations.



# What do the TEKS say?

- Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, number sense, and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

# Grade by Grade

- Grade K, Standard 2(B)
  - read, write, and represent whole numbers from 0 to at least 20 with and without objects or pictures
- Grade K, Standard 3(C)
  - explain the strategies used to solve problems involving adding and subtracting within 10 using spoken words, concrete and pictorial models, and number sentences.
- Grade K, Standard 8
  - (B) use data to create real-object and picture graphs; and
  - (C) draw conclusions from real-object and picture graphs.

# Grade by Grade

- Grade 1, Standard 2(C)
  - use objects, pictures, and expanded and standard forms to represent numbers up to 120
- Grade 1, Standard 3
  - (C) compose 10 with two or more addends with and without concrete objects
  - (E) explain strategies used to solve addition and subtraction problems up to 20 using spoken words, objects, pictorial models, and number sentences
- Grade 2, Standard 3(C)
  - use concrete models to count fractional parts beyond one whole using words and recognize how many parts it takes to equal one whole
- Grade 2, Standard 6(A)
  - model, create, and describe contextual multiplication situations in which equivalent sets of concrete objects are joined

# Grade by Grade

- Grade 3, Standard 3
  - (A) represent fractions greater than zero and less than or equal to one with denominators of 2, 3, 4, 6, and 8 using concrete objects and pictorial models, including strip diagrams and number lines
  - (F) represent equivalent fractions with denominators of 2, 3, 4, 6, and 8 using a variety of objects and pictorial models, including number lines
- Grade 3, Standard 4(K)
  - solve one-step and two-step problems involving multiplication and division within 100 using strategies based on objects; pictorial models, including arrays, area models, and equal groups; properties of operations; or recall of facts

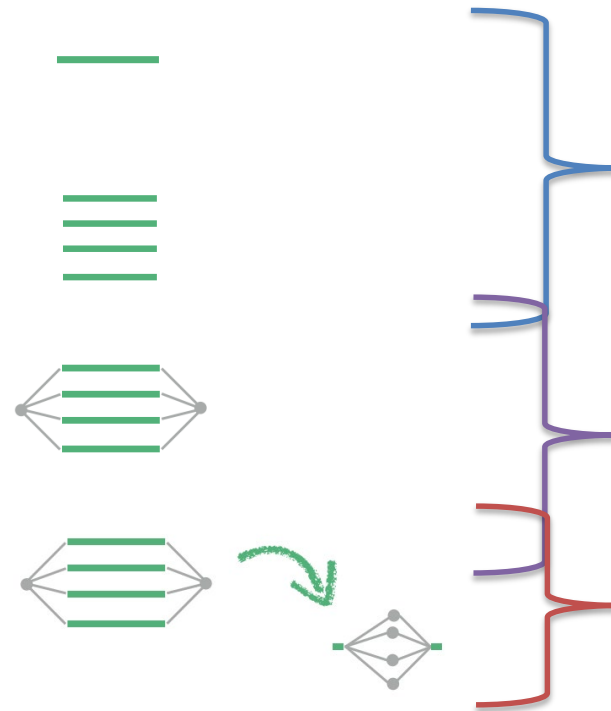
# Grade by Grade

- Grade 4, Standard 2(E)
  - represent decimals, including tenths and hundredths, using concrete and visual models and money
- Grade 4, Standard 3(B)
  - decompose a fraction in more than one way into a sum of fractions with the same denominator using concrete and pictorial models and recording results with symbolic representations
- Grade 5, Standard 3(H)
  - represent and solve addition and subtraction of fractions with unequal denominators referring to the same whole using objects and pictorial models and properties of operations
- Grade 5, Standard 4(G)
  - use concrete objects and pictorial models to develop the formulas for the volume of a rectangular prism, including the special form for a cube ( $V = l \times w \times h$ ,  $V = s \times s \times s$ , and  $V = Bh$ )

# What is deep learning?

## SOLO Taxonomy

- **Uni-structural Ideas**
  - individual idea in isolation
- **Multi-structural Ideas**
  - multiple ideas, each in isolation
- **Relational Ideas**
  - integrating ideas into a structure
- **Extended Abstract Ideas**
  - generalize to a new domain



## Phases of Learning

Surface Learning

Deep Learning

Transfer Learning



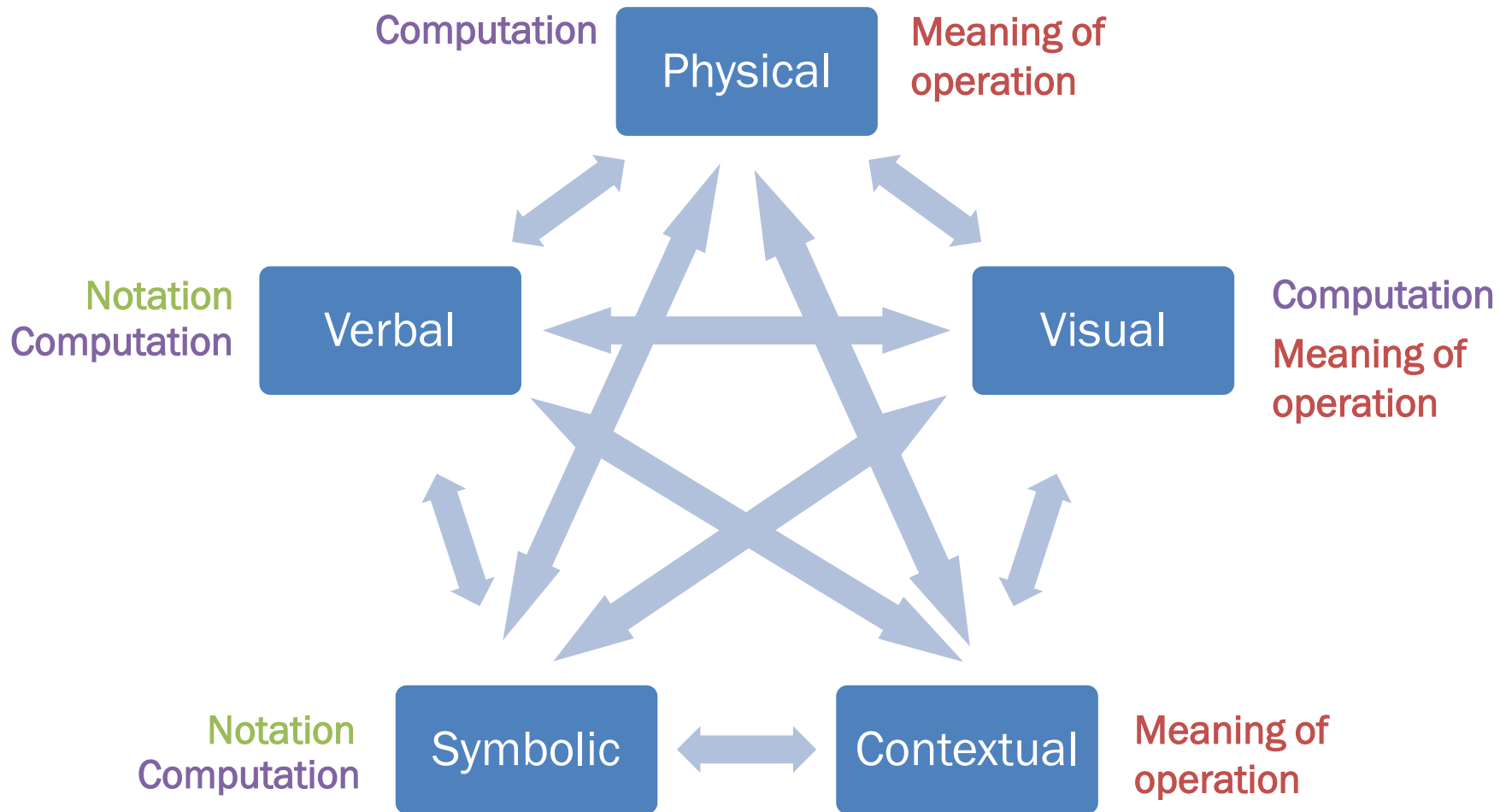
# Number Sense (Mix and Match Cards)

- Form 2 groups at your table & sort the cards by color.
- Your 8 cards represent 2 numbers – sort them into these two groups.
- Identify each digit in the value in each of the representations – how does this appear in each representation?
- Which representation(s) are most helpful to compare values? To add or subtract? To see the magnitude of the number?
- Which representation(s) might be most confusing to your students? Why? How do the other representations help overcome this?

# Subtraction

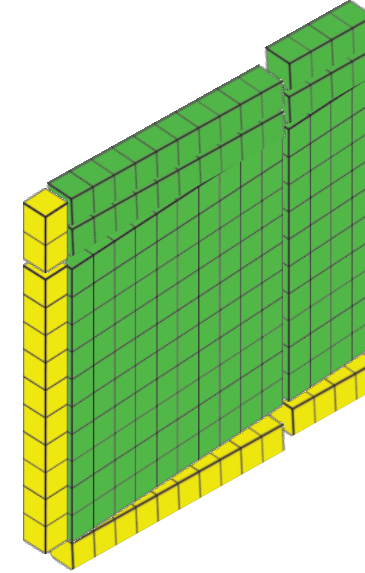
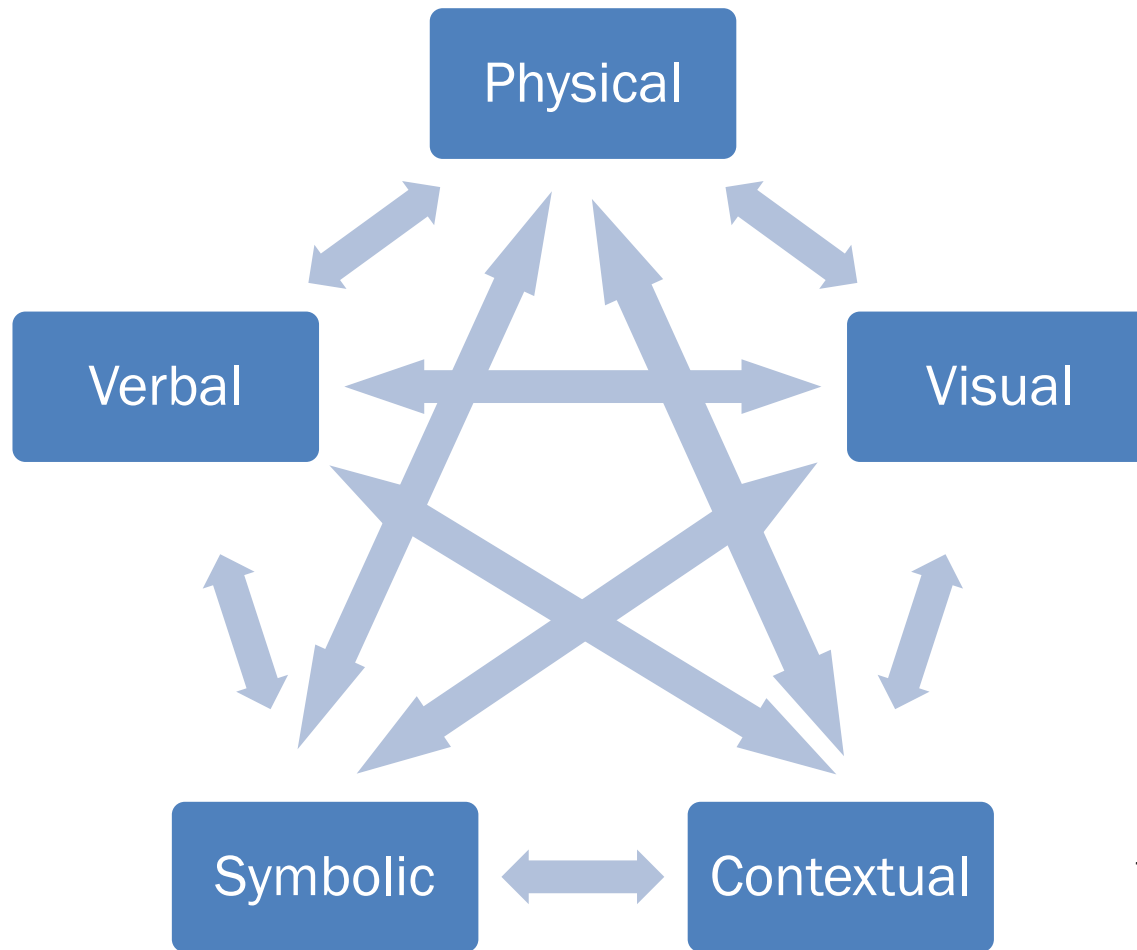
- What does deep learning for subtraction look like?
- About the operation (meaning)
  - What kinds of problems can subtraction help us solve?
  - When do we use subtraction?
- About computation
  - What strategies and procedures help us subtract?
  - How does this change for different categories of numbers?
- About notation
  - How do we represent subtraction?

# What do we learn where?



# Multi-digit Multiplication

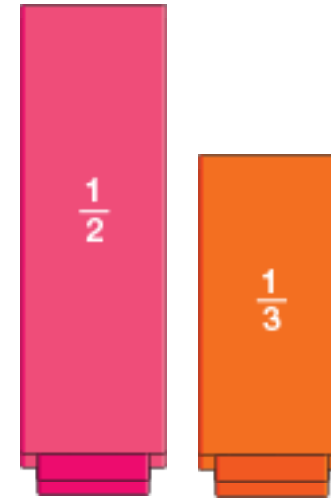
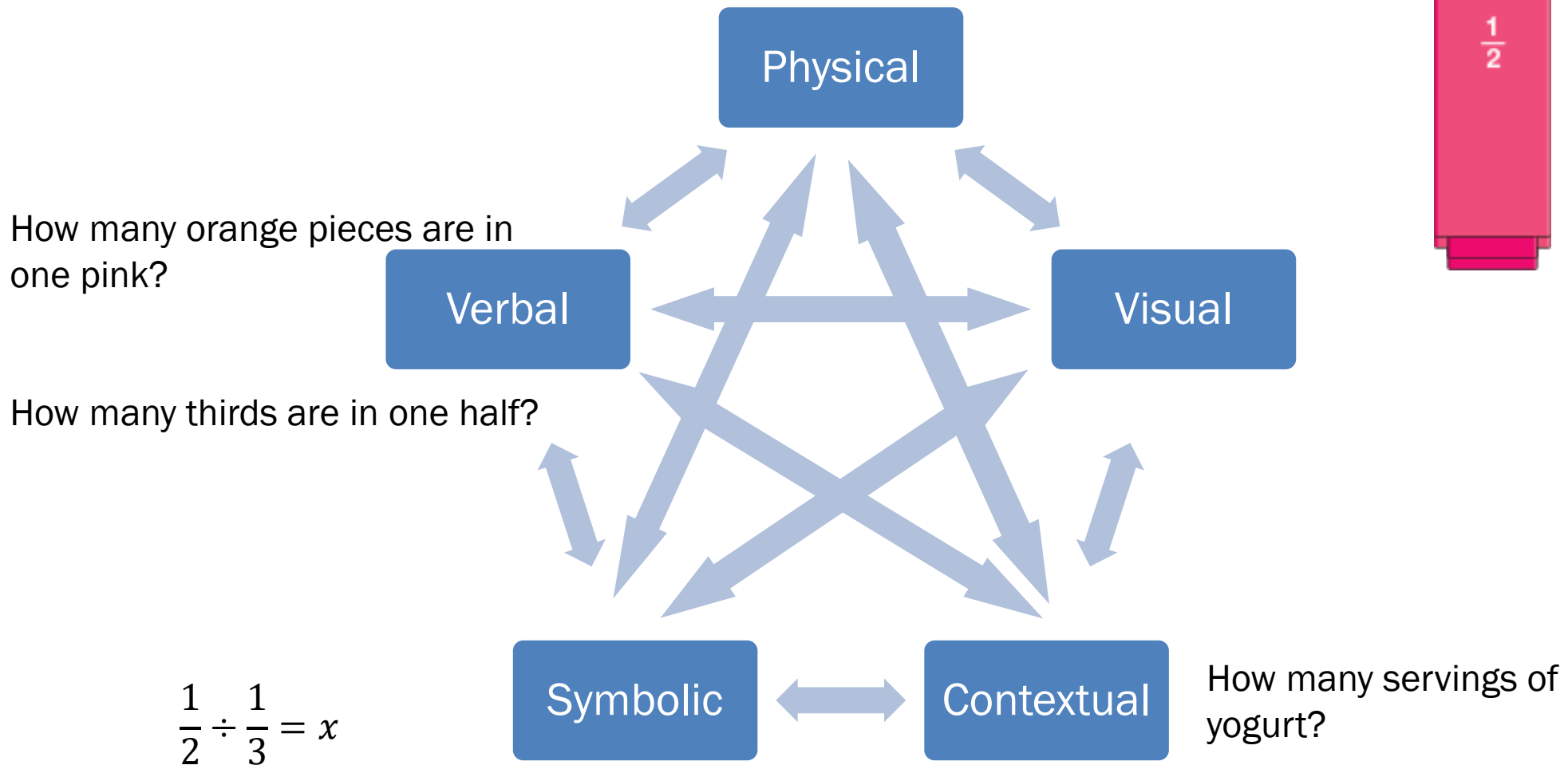
$$\begin{array}{r}
 14 \\
 \times 12 \\
 \hline
 8 \\
 20 \\
 40 \\
 + 100 \\
 \hline
 168
 \end{array}$$



2	20	8
+		
10	100	40
	10	+ 4


Hannah is moving into a larger bedroom. Her new room is 12 feet wide by 14 feet long. How many carpet tiles will Hannah need for the floor of her new room?

# Fraction by Fraction Division



# Closing Thoughts

- Deep learning is rich and broad – focused on connections.
- Connections can develop by exploring the same concept through a variety of representations.
- How do we spend more time in deep learning?



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