When my 2+3 isn't the same as your 2+3

Sara Delano Moore
sara@sdmlearning.com
@saradelanomoore

Kimberly Morrow-Leong
morrowmath@gmail.com
@kmorrowleong

with Linda M. Gojak

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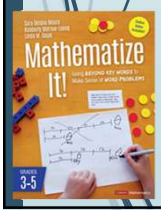
Operation Sense

Knowing and applying the full range of work for mathematical operations (for example, addition, subtraction, multiplication, and division).



Number sense and operation sense are separate but complementary ideas.

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Operation Sense

- **Working Models of Operations**
  - Understand and use a wide variety of models of operations beyond the basic and intuitive models of operations.
- **Representations of Operations**
  - Use appropriate representations of actions or relationships strategically.
- **Mathematizing**
  - Can mathematize a situation, translating a contextual understanding into a variety of other mathematical representations.
- **Number Categories**
  - Apply their understanding of operations to any quantity, regardless of the class of number.

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This is not about computation

Computational Strategies

- Counting on or back
- Doubles
- Bridging ten
- Known related facts
- These are strategies students use to compute the answer, AFTER students understand the situation.

Models of Operations

- Operation Sense is about describing what is happening in the situation. What models and representations show the action or relationships in the problem?
- Operation sense comes BEFORE students select a computation strategy to find the solution.

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47% OF WORD PROBLEM ERRORS OCCUR BEFORE A CALCULATION PROCESS TAKES PLACE.

(Newman, 1977, as cited in Watson, 1980)

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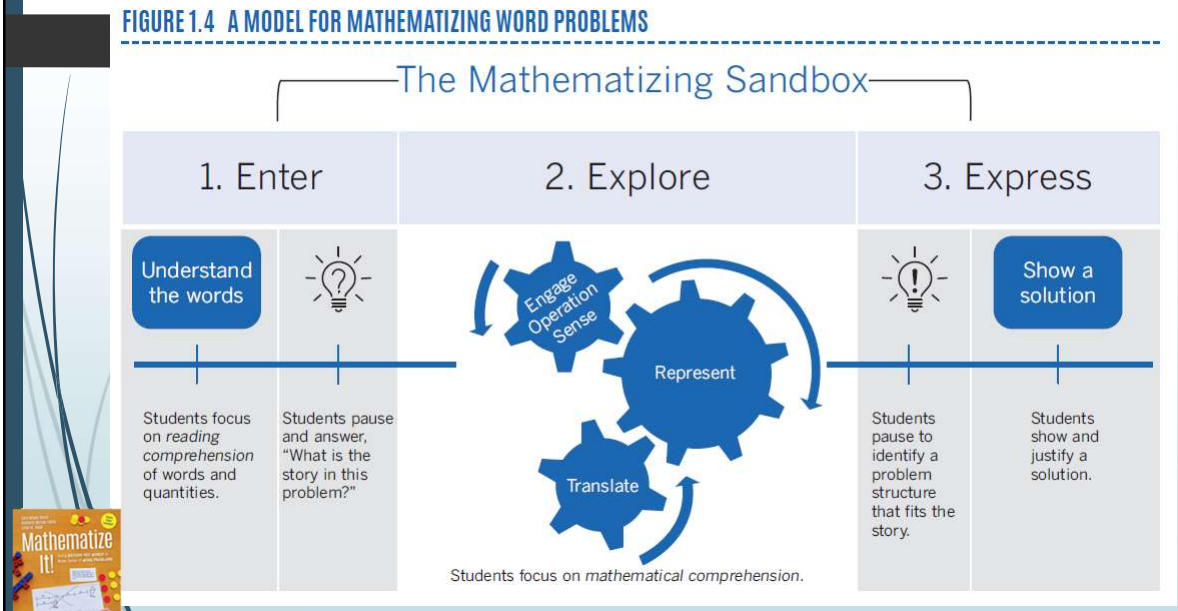
FIGURE 1.4 A MODEL FOR MATHEMATIZING WORD PROBLEMS



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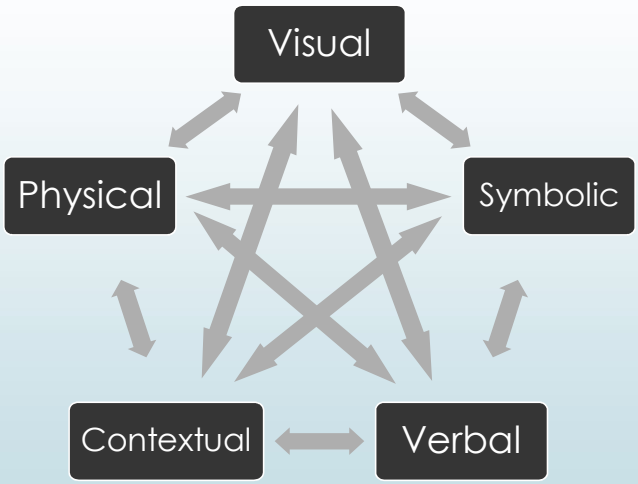
Use & Connect multiple representations



Leinwand, S., Brahier, D. J., & Huinker, D. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: National Council of Teachers of Mathematics.

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How are these problems different?

There were 9 eggs in the carton. Emily ate two eggs for her breakfast. How many eggs are left in the carton?

There are 9 children on the soccer team. Two of the children are wearing green shoes. The rest of the children are wearing white shoes. How many children are wearing white shoes?

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Actions vs Relationships

Action Situations

- Something happens.
- The problem tells a story with a beginning, middle, and end.
- "Things" come into or leave a situation.

Relationship Situations

- Nothing happens.
- In Part-Part-Whole situations, there are groups or categories of "things."
- Comparisons are another kind of relationship situation.

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Two different relationships

Part-Part-Whole

- Nothing happens.
- There's a group of some sort.
- The problem is about parts of the group.

Additive Comparison

- Nothing happens.
- There are two groups of some sort.
- The problem is about more and less – the difference between the two groups.

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WHAT'S HAPPENING?

- Is there action?
  - Are things coming or going?
- Are we grouping/regrouping?
- Are we comparing?

WHAT INFORMATION IS MISSING?

START ± CHANGE = RESULT

PART + PART = WHOLE

SMALLER	DIFFERENCE
LARGER	

Addition and Subtraction Problem Situations

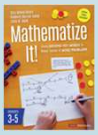
ACTIVE SITUATIONS

	Result Unknown	Change Addend Unknown	Start Addend Unknown	
Add-To	Paulo counted out 75 crayons and put them in the basket. Then he found 23 more crayons under the table. He added them to the basket. How many crayons are now in the basket? $75 + 23 = x$ $23 = x - 75$	Paulo counted out 75 crayons and put them in the basket. Then he found some more crayons under the table. He added them to the basket and now there are 98 crayons in the basket. How many crayons were under the table? $75 + x = 98$ $x = 98 - 75$	Paulo was organizing the crayons at his table. He found 23 crayons under the table and added them to the basket. When he counted, there were 98 crayons in the basket. How many crayons were in the basket before Paulo looked under the table for crayons? $x + 23 = 98$ $98 - 23 = x$	
Take-From	There are 26 students in Mrs. Amadi's class. After lunch, 15 left to get ready to play in the band at the assembly. How many students are not in the band? $26 - 15 = x$ $26 - 15 + x$	There are 26 students in Mrs. Amadi's class. After the band students left the classroom for the assembly, there were 11 students still in the classroom. How many students are in the band? $26 - x = 11$ $x + 11 = 26$	After lunch, 15 band students left Mrs. Amadi's class to get ready to play in the assembly. There were 11 students still in the classroom. How many students are in Mrs. Amadi's class? $x - 15 = 11$ $15 + 11 = x$	

RELATIONSHIP (NONACTIVE) SITUATIONS

	Total Unknown	One Part Unknown	Both Parts Unknown
Part-Part-Whole	The 4th grade held a vote to decide where to go for the annual field trip. 32 students voted to go to the ice skating rink. 63 voted to go to the local park. How many students are in the 4th grade? $32 + 63 = x$ $x = 63 - 32$	The 4th grade held a vote to decide where the 95 students in the grade should go for their annual field trip. 32 students voted to go to the ice skating rink. The rest chose the local park. How many voted to go to the park? $32 + x = 95$ $x = 95 - 32$	The 4th grade held a vote to decide where the 95 students in the grade should go for their annual field trip. Some students voted to go to the ice skating rink and others voted to go to the local park. What are some possible combinations of votes? $x + y = 95$ $95 - x = y$

	Difference Unknown	Greater Quantity Unknown	Lesser Quantity Unknown	
Additive Comparison	Jessie and Roberto both collect baseball cards. Roberto has 71 cards and Jessie has 53 cards. How many fewer cards does Jessie have than Roberto? $53 + x = 71$ $71 - 53 = x$	Jessie and Roberto both collect baseball cards. Jessie has 53 cards and Roberto has 18 more cards than Jessie has. How many baseball cards does Roberto have? $53 + 18 = x$ $x - 18 = 53$	Jessie and Roberto both collect baseball cards. Roberto has 71 cards and Jessie has 18 fewer cards than Roberto. How many baseball cards does Jessie have? $71 - 18 = x$ $x + 18 = 71$	



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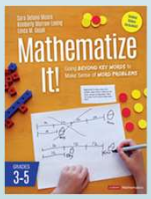
Strategies

- Create multiple representations with a variety of tools.
- Use numberless problems to help students slow computation.
- Think about what is happening in the problem.
  - Is it about action or relationships?
  - What is changing?
  - Are we grouping and regrouping?
  - Are we comparing?
- Use literature to introduce situations.



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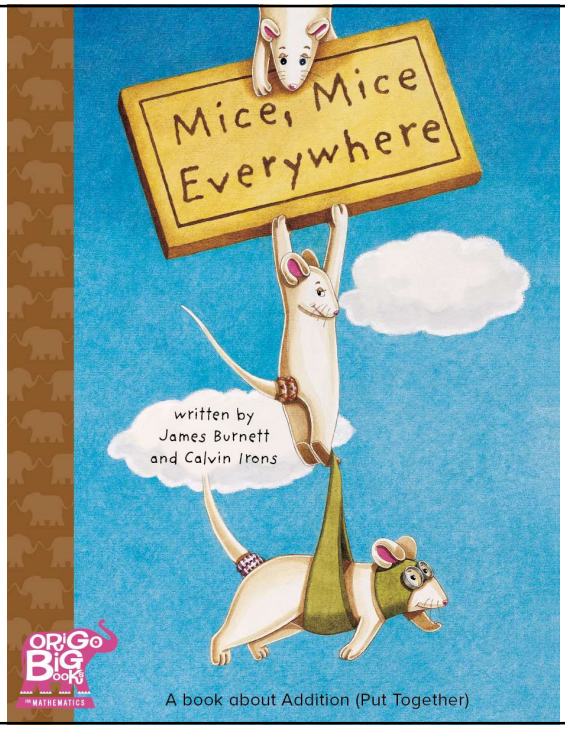

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Closing thoughts

- Operation sense is about context, not computation.
- Pause to mathematize when working with word problems.
- Spend time in the *Mathematizing Sandbox*.
- Use models and representations of operations and pause to highlight mathematical structure.
- Word problems are a tool, not the end game.

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