A Clear Vision for Utilizing Number Lines

Debi DePaul

PROFESSIONAL LEARNING
www.origoeducation.com

ORIGO EDUCATION

A Clear Vision for Utilizing Number Lines

For more information visit www.origoeducation.com

Build a Number Track

- Put a finger on 5, put another finger on 7. What number comes between 5 and 7?
- Break apart all the cubes and put the numbers in order.
- What number comes just after five?

ORIGO EDUCATION

Build a Number Track

- Take a cube away and ask a friend to name the missing number.
- What number do you land on if you start at 5 and jump on 2 more?
- Turn over every second cube. Read the numbers.

ORIGO EDUCATION

Slides and Ladders

2 to 4 players

Adding on a number track

Purpose

In this race game, the students add 1, 2, 3, 4, 5, or 6 to one- and two-digit numbers. The students are aided by a number track, which is provided as a game board. The students will be encouraged to vary their mental strategy depending on the number being added.

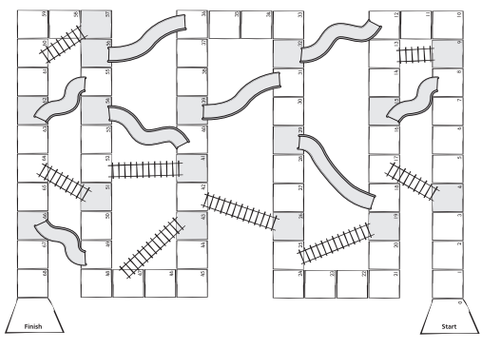
Materials

Each group of players will need

- A 'Slides and Ladders' game board. Copy pages 62 and 63, and follow the directions to make the board shown below. The finished game can be laminated for durability.
- One (1) standard number cube showing numerals or dot patterns 1-6.

Each player will need

- One (1) counter (a different color for each player).



How to Play

The aim is to reach the finish line.

- Each player selects a counter and places it at zero.
- The first player rolls the number cube, adds the number to zero and moves his or her counter to the resulting sum.
- Each of the other players has a turn.
- If a player's counter finishes on the base of a ladder, the player must move his or her counter up to the top of the ladder.
- If a player's counter finishes at the top of a slide, the player must move his or her counter down to the bottom of the slide.
- More than one counter can occupy a space at one time.
- The first player to go beyond the finish line is the winner.

Reading the Research

Observing students engaged in games and problem-solving activities can yield rich information about their fact knowledge. For example, as students play a game, a teacher may notice whether they are using counting strategies, derived-fact strategies, or known facts (Isaacs & Carroll, 1999).

Before the Game

Explain the rules to three students. Invite them to demonstrate how to play to the remainder of the class. During the game, encourage the players to verbalize each addition sentence. For example, if a player's counter is on 12 and 3 is rolled, the player should say, *12 plus 3 is 15.*

During the Game

Identify the strategies that players use to move their counters along the game board. Some players will use more efficient strategies than others. For example, one player may use a count-on strategy for every turn. Another player may only count on if the number being added is small. When adding bigger numbers, such as 5 or 6, some players may choose to break the number into more manageable parts. For example, if starting at 18 and adding 6, a player may think, *18 plus 2 is 20 plus 4 more is 24.* This strategy is particularly helpful when bridging over a ten. Encourage the students to share their strategies. Sometimes, students will change their own method when they see and hear their friends using methods that appear easier.

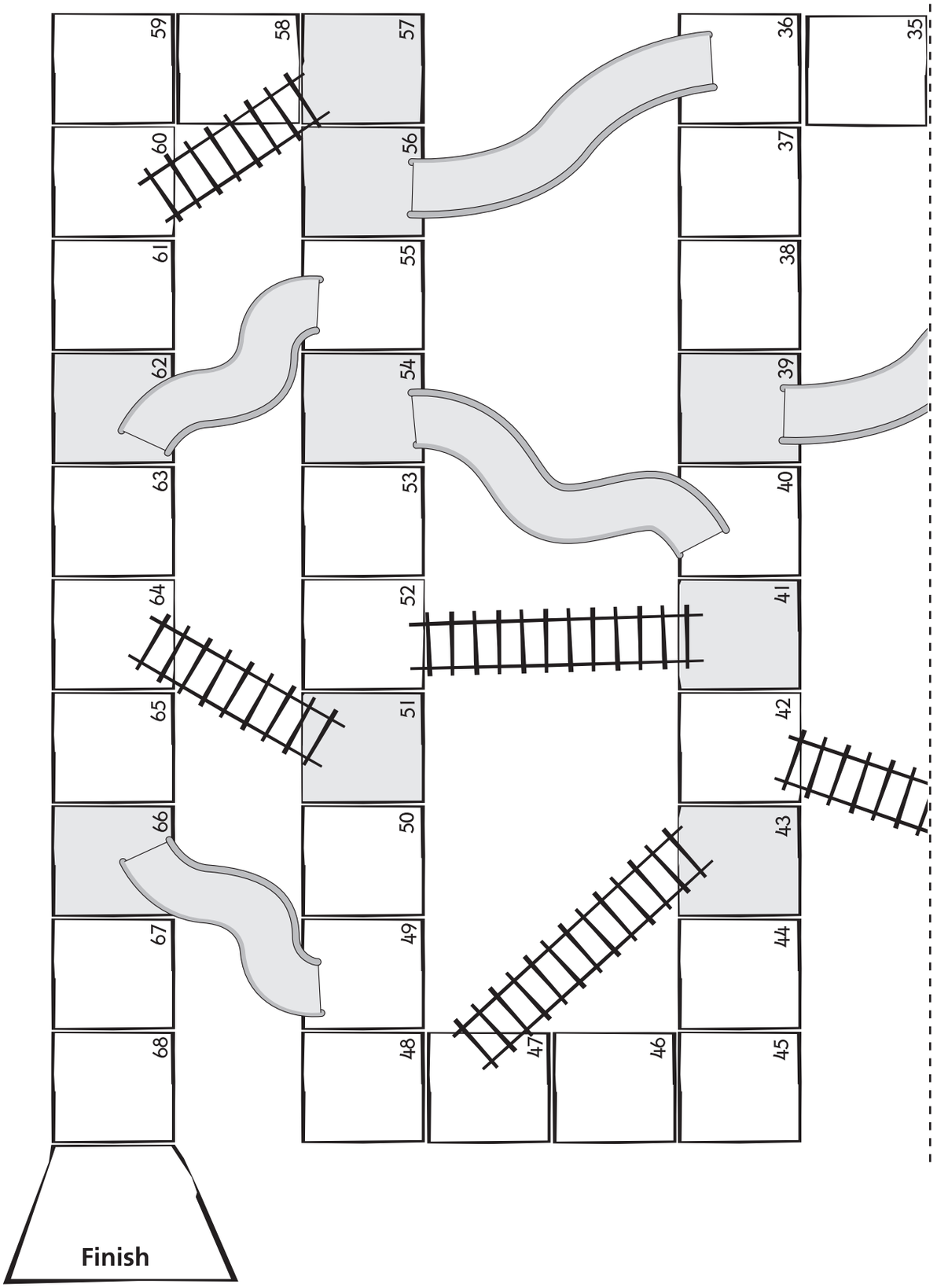
After the Game

The number track allows for the use of many different strategies. Lead a discussion about the various methods the students use to add. Say, ***Suppose you were positioned on 12 and you rolled 6. How would you figure out where to move?*** Elicit a variety of responses. Some students will count on six. Others may jump along in steps of two or three. Some students may simply know that 12 + 6 = 18. Request the attention of those students who used a count-on strategy. Ask, ***How would you add 6 if you were positioned on a number closer to a ten, such as 28 or 29?*** If they count on again, challenge the group to share another way that it could be done.

Beyond the Game

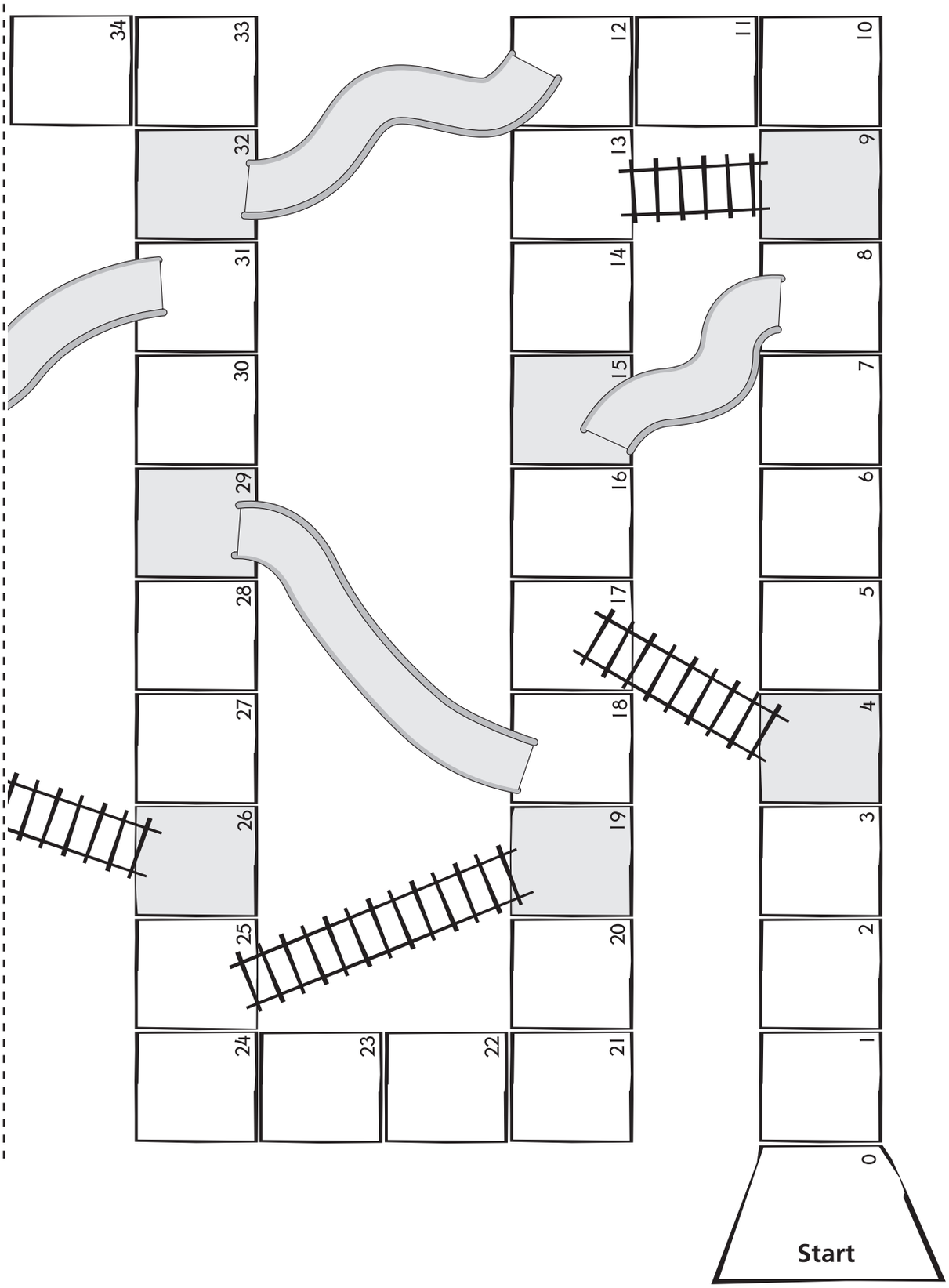
- Control the number being added by changing the number cube. For example, writing 1, 1, 2, 2, 3, and 3 on the faces of a blank cube will allow the students to practice counting on 1,2, or 3.
- Use the cube described above to play a subtraction version of the game. In this game, the players start at the finish line and play backwards.

Slides and Ladders



Paste the edge of page 63 here.

Slides and Ladders



Cut along this line and attach to page 62.

First to 100

Adding on a hundred chart

2 players

Purpose

In this game, students practice adding two-digit numbers by breaking up one addend into tens and ones. A hundred chart is used to help the students add the tens and then the ones. For example, to calculate 43 + 24, the students are encouraged to think, 43 + 20 + 4.

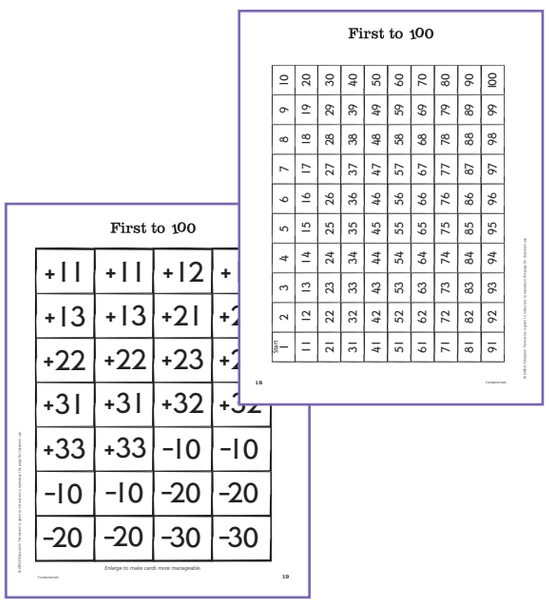
Materials

Each pair of players will need

- A 'First to 100' game board (page 18) as shown below.
- One (1) set of operation cards. Copy page 19 as shown below. Cut out and laminate the cards to make one set.

Each player will need

- One (1) transparent counter (a different color for each player).



How to Play

The aim is to reach or pass 100 on the game board.

- The operation cards are shuffled and placed face down in a stack.
- The players place their counters in the 'Start' position (1) on the game board.
- The first player draws the top card. If an addition card is drawn, the player moves his or her counter that number of spaces forward on the game board.

  Example: Katie draws '+ 12' and moves her counter down one row (+ 10) and two spaces to the right (+ 2).

- Initially, if a subtraction card is drawn, the player remains at 'Start'. Throughout the game, if a subtraction card is drawn, the player moves his or her counter back that number of spaces.

  Example: Jason draws '– 20' and moves his counter up two rows (– 20).

- If a subtraction card would result in a score less than 1, the player returns his or her counter to 'Start'.
- The card is placed at the bottom of the stack.
- The other player has a turn.
- The first player to reach or pass 100 is the winner.

Reading the Research

Hundred boards can be a useful model for helping children see how higher-decade addition and subtraction facts are related. They also help children see patterns and relationships. For example, 8 + 7 begins at 8 and ends on 15 on the next row. Similarly, the sums 38 + 7, 48 + 7, … 88 + 7 all begin and end in the same columns (Van de Walle & Bowman Watkins, 1993).

Before the Game

Show a transparency of the game board on the overhead projector. Place a transparent counter on 54. Select an addition card such as '+ 32' and ask students to figure out the sum. Encourage them to explain how they calculated the answer. For example, can the students see that they need to move the counter down three rows and two spaces to the right? Do they know that this is the same as adding 30 and then adding 2? Discuss alternative strategies, such as adding the ones first, followed by the tens.

During the Game

At various stages of the game, ask students how many more they need to make 100. Encourage them to share how they figured out the answer. The discussion will vary, but they should see that one efficient strategy is to build the ones to the next multiple of ten and then figure out the number of tens they need to get to 100. For example, if a player has a counter on 62, he or she will need 8 more to make 70 and 30 more to make 100, so the player needs a total of 38. Alternatively, they could count forward in tens and then build the ones to 100.

After the Game

Ask students to explain a general method of finding out how many more they need to make 100. A student might say, *I figure out how many more ones I need to get to the next multiple of ten and then see how many more tens I need to get to 100*.

Use a transparency of the game board on the overhead projector to pose hypothetical questions. For example, place a counter on the number 83 and ask what the total would be if '+ 23' was drawn from the stack of cards. Can the students see that the same strategies work for moves beyond 100?

Have the students calculate similar sums without the aid of the game board. Ask them to explain their methods. For example, write 51 + 42 on the board and ask how they could calculate the answer. A student might say, *51 + 40 = 91 and 91 + 2 = 93*. Can the students see that this is a much more efficient strategy than counting by ones?

Beyond the Game

Give the students square grid paper to make a game board for numbers beyond 100, such as 301-400. Using the operation cards from 'First to 100', the students can race to a new target, for example 400. The game rules remain the same.

First to 100

Start 1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

First to 100

+11	+11	+12	+12
+13	+13	+21	+21
+22	+22	+23	+23
+31	+31	+32	+32
+33	+33	−10	−10
−10	−10	−20	−20
−20	−20	−30	−30

Enlarge to make cards more manageable.

Break it Down

Adding on a number chart

2 players

Purpose

In this game, students practice adding two-digit numbers by breaking up one addend into tens and ones. A partially-filled number chart is used to help the students add the tens and then the ones. For example, to calculate 443 + 24, the students are encouraged to think, 443 + 20 + 4.

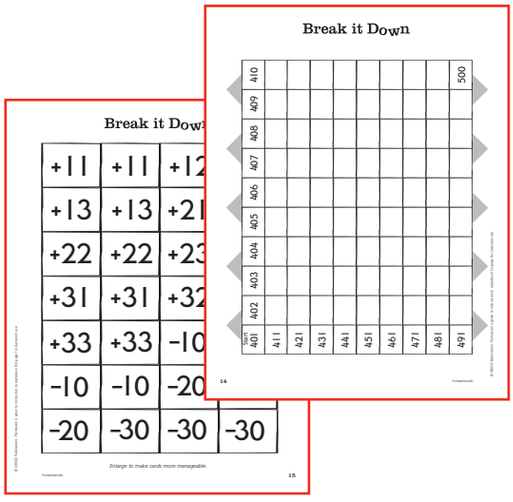
Materials

Each pair of players will need

- A 'Break it Down' game board (page 14) as shown below.
- One (1) set of operation cards. Copy page 15 as shown below. Cut out and laminate the cards to make one set.

Each player will need

- One (1) transparent counter (a different color for each player).



How to Play

The aim is to reach or pass 500 on the game board.

- The operation cards are shuffled and placed face down in a stack.
- The players place their counters in the 'Start' position (401) on the game board.
- The first player draws the top card. If an addition card is drawn, the player moves his or her counter that number of spaces forward on the game board.

  Example: Michelle draws '+ 32' and moves her counter down three rows (+ 30) and two spaces to the right (+ 2).

- Initially, if a subtraction card is drawn, the player remains at 'Start'. Throughout the game, if a subtraction card is drawn, the player moves his or her counter back that number of spaces.

  Example: James draws '– 30' and moves his counter up three rows (– 30).

- If a subtraction card would result in a score less than 401, the player returns his or her counter to 'Start'.
- The card is placed at the bottom of the stack.
- The other player has a turn.
- The first player to reach or pass 500 is the winner.

Reading the Research

Although more abstract, a partially-filled hundred chart works better to activate mental strategies than the easier filled-in chart that is often used in classrooms (Beishuizen, 1993).

Before the Game

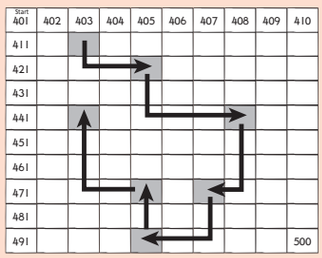
Show a transparency of the game board on the overhead projector. Place a transparent counter on 423. Select an addition card, such as '+ 31', and ask students to figure out the sum. Encourage them to explain their thinking. Can the students see they need to move the counter down three rows and one space to the right? Do they know this is the same as adding 30, then adding 1? Discuss alternative strategies, such as adding the ones first, then the tens.

During the Game

At various stages of the game, ask students how many more they need to make 500. Encourage them to share how they figured out the answer. The discussion will vary, but they should see that one efficient strategy is to build the ones to the next multiple of ten and then figure out the number of tens they need to get to 500. For example, if a player has a counter on 472, he or she will need 8 more to make 480 and 20 more to make 500, so the player needs a total of 28. Alternatively, they could count forward in tens and then build the ones to 500.

After the Game

Draw arrows on an overhead transparency of the game board as shown. Tell the students that the arrows represent numbers that have been added or subtracted. The shaded areas indicate the starting numbers and the answers. Have the students figure out the numbers that have been added or subtracted each time.



Afterwards, give the students a copy of the game board. As cards are drawn, they could draw arrows to show how to add or subtract the two-digit number, before shading the box that would show the answer.

-11	-11	-12	-12
-13	-13	-21	-21
-22	-22	-23	-23
-31	-31	-32	-32
-33	-33	+10	+10
+10	+10	+20	+20
+20	+30	+30	+30

Beyond the Game

- The students can play the same game using different target numbers, such as 200, 300, or 450. They will need to make a new game board for each new target number. The same set of operation cards can be used.
- The students could play a subtraction version of 'Break it Down'. Make a new set of cards as shown. In this game, the students start on 500 and race backwards to 401.

Break it Down

410									500
409									
408									
407									
406									
405									
404									
403									
402									
Start 401	411	421	431	441	451	461	471	481	491

Break it Down

+11	+11	+12	+12
+13	+13	+21	+21
+22	+22	+23	+23
+31	+31	+32	+32
+33	+33	−10	−10
−10	−10	−20	−20
−20	−30	−30	−30

Enlarge to make cards more manageable.

Leaps and Bounds

Using number lines to calculate difference

2 or 3 players

Purpose

In this game, students calculate the difference between a pair of two-digit numbers. The numbers do not require the students to bridge across a ten.

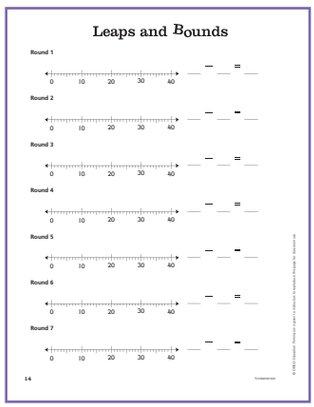
Materials

Each group of players will need

- Two (2) number cubes made from blank wooden cubes. One cube (Cube A) should show the numerals 25, 27, 29, 35, 36, and 38. The second cube should show numerals 10-15.

Each player will need

- A 'Leaps and Bounds' game board (page 14) as shown below.


Leaps and Bounds

Round 1

Round 2

Round 3

Round 4

Round 5

Round 6

Round 7

How to Play

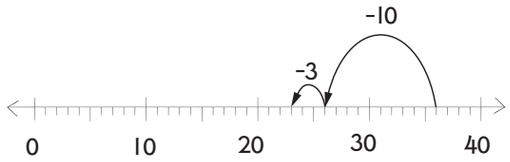
The aim is to achieve the greater difference.

- The first player rolls the number cubes.
- Using the two numbers, the player writes a subtraction sentence in the first round on his or her game board.

  Example: Beth rolls 13 and 36, and writes 36 – 13 = ___.

- The player mentally calculates the answer and writes it in the number sentence, then uses the number line to show his or her thinking.

  Example: Beth writes 36 – 13 = 23 and shows how she starts with 36, counts back 10, then 3.



- The other player(s) has a turn.
- The player with the greater difference (answer) scores a point for the round. This is indicated with a ✓.
- The player with the greater number of points after seven rounds is the winner.

Reading the Research

Research shows that children need many different kinds of number experiences to help them deal with real-life situations. It is important that they work regularly with a variety of number representations, including a counting model, a linear model such as a number line, a place value model and a quantity model (Irons, 2002).

Before the Game

Draw a number line on the board, like the ones shown on page 12. Invite a volunteer to roll the number cubes and write a subtraction sentence. Encourage the students to picture in their heads the jumps they would make to calculate the answer. Call upon one student to draw the jumps and explain his or her strategy. Repeat this activity several times before explaining the rules of the game.

During the Game

Observe the various strategies that different students use to calculate the answers. At a later stage, call upon these students to share their methods with the entire class. For example, to calculate 29 – 14, the students may count back 10 then count back 4, or vice versa. Alternatively, they may change the two numbers by thinking, 29 – 14 is the same as 30 – 15.

After the Game

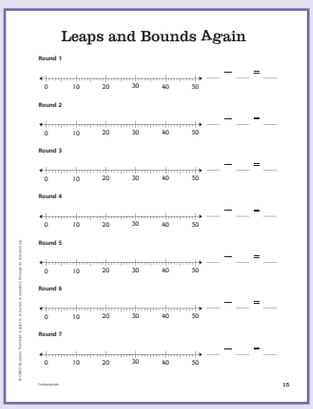
Draw another number line on the board. Share stories that involve all three models of subtraction. At this stage, make sure they do not involve bridging across a ten, for example

Take away: *A school bus was carrying 37 students. Twelve students got off. How many students were left on the bus?* (25 students.)

Missing addend: *Jemma needs $29 to buy a gift. She has saved $14. How much more money does she need to save?* ($15.)

Comparison/difference: *Andy planted 26 trees in the morning and 15 trees in the afternoon. How many more trees did he plant in the morning than the afternoon?* (11 more trees.)

For each problem, invite volunteers to draw jumps on the number line to explain how they would calculate the answer mentally. Did all students use the same strategy for all three models of subtraction? Did any one strategy better suit a particular model?



Beyond the Game

- Extend the game by changing Cube A. Make a new cube using the numerals 37, 38, 39, 45, 47, and 49. Each player will also need a copy of the 'Leaps and Bounds Again' game board on page 15 (illustrated).
- Change Cube A again to show the numerals 22, 23, 24, 31, 32, and 33. This will require the students to bridge across a ten. Either game board can be used.

Leaps and Bounds

Round 1

— — = —

Round 2

— — = —

Round 3

— — = —

Round 4

— — = —

Round 5

— — = —

Round 6

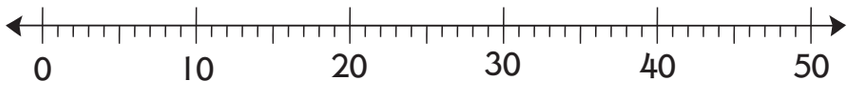
— — = —

Round 7

— — = —

Leaps and Bounds Again

Round 1

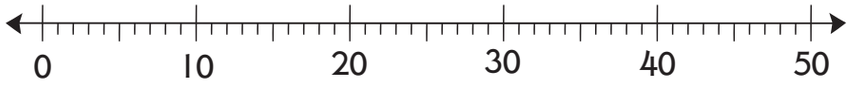


— — = —

Round 2

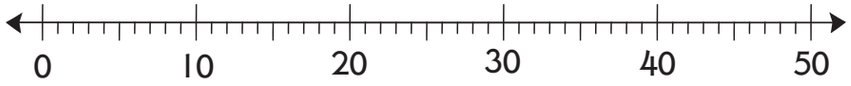
— — = —

Round 3



— — = —

Round 4

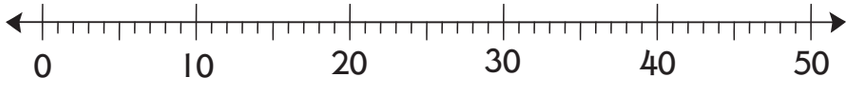


— — = —

Round 5

— — = —

Round 6



— — = —

Round 7

— — = —

Jump Back

Using number lines to calculate difference

2 or 3 players

Purpose

In this game, students calculate the difference between two- and three-digit numbers. The numbers involved do not require the students to bridge across a ten or hundred. The extension activity requires the students to subtract decimal fractions involving tenths.

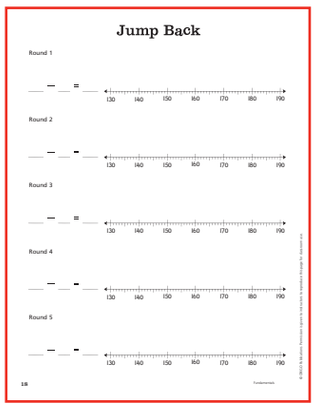
Materials

Each group of players will need

- Two (2) number cubes made from blank wooden cubes. One cube should show the numerals 166, 167, 177, 178, 188, and 189. The second cube should show the numerals 23, 24, 25, 33, 34, and 35.

Each player will need

- A 'Jump Back' game board (page 18) as shown below.



How to Play

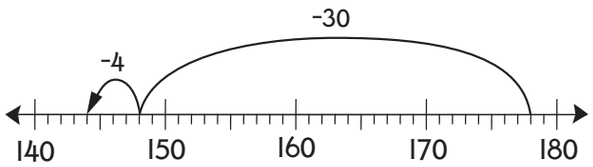
The aim is to achieve the greater difference.

- The first player rolls the number cubes.
- Using the two numbers, the player writes a subtraction sentence in the first round on his or her game board.

  Example: Silas rolls 34 and 178. He writes 178 – 34 = ___.

- The player mentally calculates the answer and writes it in the number sentence, then uses the number line to show his or her thinking.

  Example: Silas writes 178 – 34 = 144 and shows how he starts with 178, counts back 30, then 4.



- The other player(s) has a turn.
- The player with the greater difference (answer) scores a point for the round. This is indicated with a ✓.
- The player with the greater number of points after five rounds is the winner.

Reading the Research

Research shows that children need many different kinds of number experiences to help them deal with real-life situations. It is important that they work regularly with a variety of number representations, including a counting model, a linear model such as a number line, a place-value model and a quantity model (Irons, 2002).

Before the Game

Draw a number line on the board, like the ones shown on page 18. Invite a volunteer to roll the number cubes and write a subtraction sentence. Encourage the students to picture in their heads the jumps they would make to calculate the answer. Call upon one student to draw the jumps and explain his or her strategy. Repeat this activity several times before explaining the rules of the game.

During the Game

Observe the various strategies that different students use to calculate the answers. At a later stage, call upon these students to share their methods with the entire class. For example, to calculate 169 – 24, the students may count back 20 then count back 4, or vice versa. Alternatively, they may change the two numbers by thinking, 169 – 24 is the same as 170 – 25.

After the Game

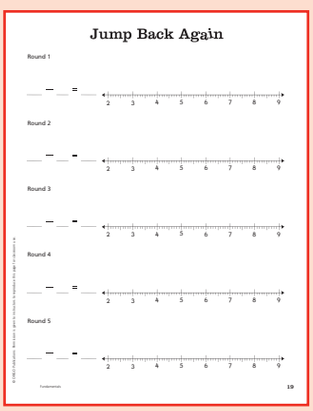
Draw another number line on the board. Share stories that involve all three models of subtraction. At this stage, make sure they do not involve bridging across a ten, for example

Take away: *A truck was loaded with 175 crates. At the first stop, 23 crates were delivered. How many crates were left on the truck?* (152 crates.)

Missing addend: *Hazel needs $188 to buy a new bike. She has saved $35. How much more money does she need to save?* ($153.)

Comparison/difference: *Grant sold 32 ice-creams before lunch and 167 ice-creams after lunch. How many more ice-creams did he sell in the afternoon than the morning?* (135 ice-creams.)

For each problem, invite volunteers to draw jumps on the number line to explain their mental strategies. Did all students use the same strategy for all three models of subtraction? Did any one strategy better suit a particular model?



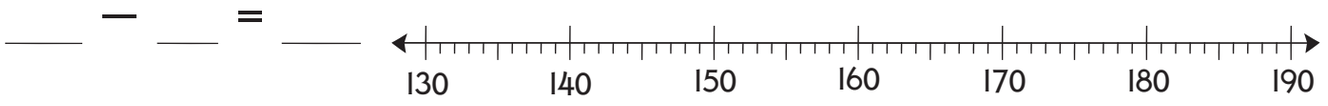
Beyond the Game

The game can be extended to involve tenths. Make two new number cubes by writing the numerals 7.7, 7.8, 7.9, 8.7, 8.8, and 8.9 on the faces of one cube and 3.2, 3.3, 4.3, 4.4, 5.4, and 5.5 on the faces of another blank cube. Each player will also need a copy of the 'Jump Back Again' game board on page 19 (illustrated). The rules of the game are the same.

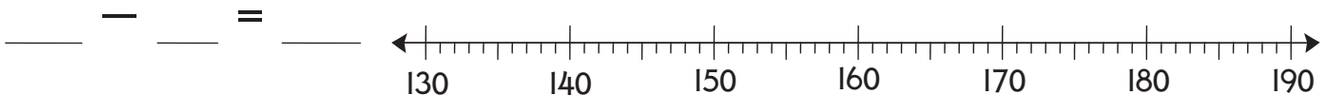
Jump Back

Round 1

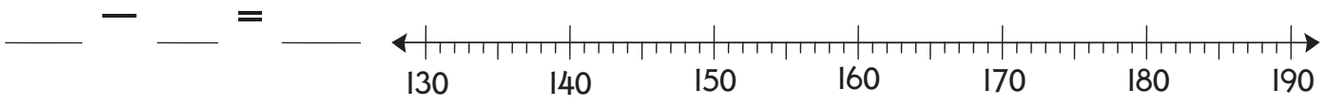
Round 2



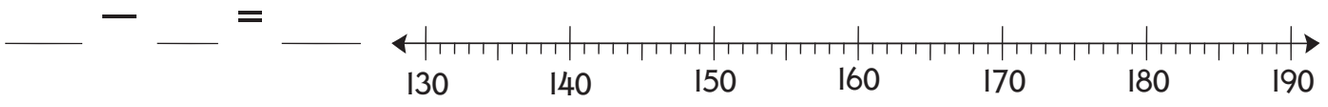
Round 3



Round 4

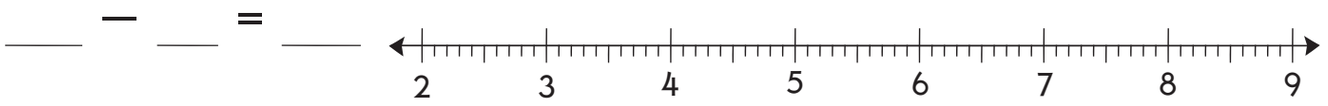


Round 5

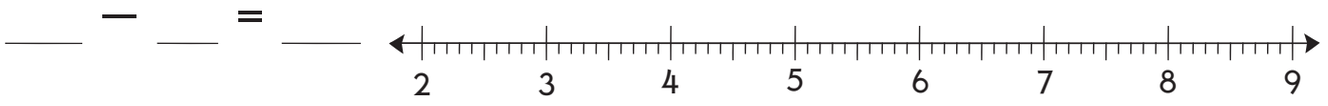


Jump Back Again

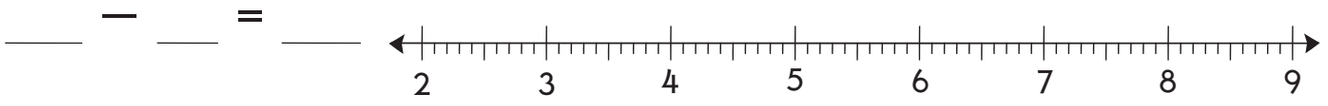
Round 1



Round 2

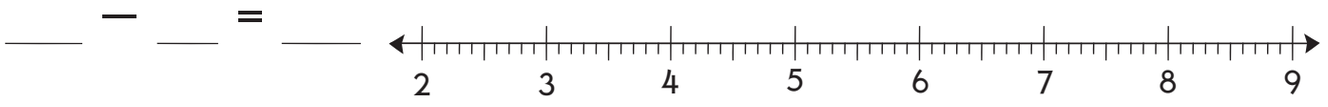


Round 3



Round 4

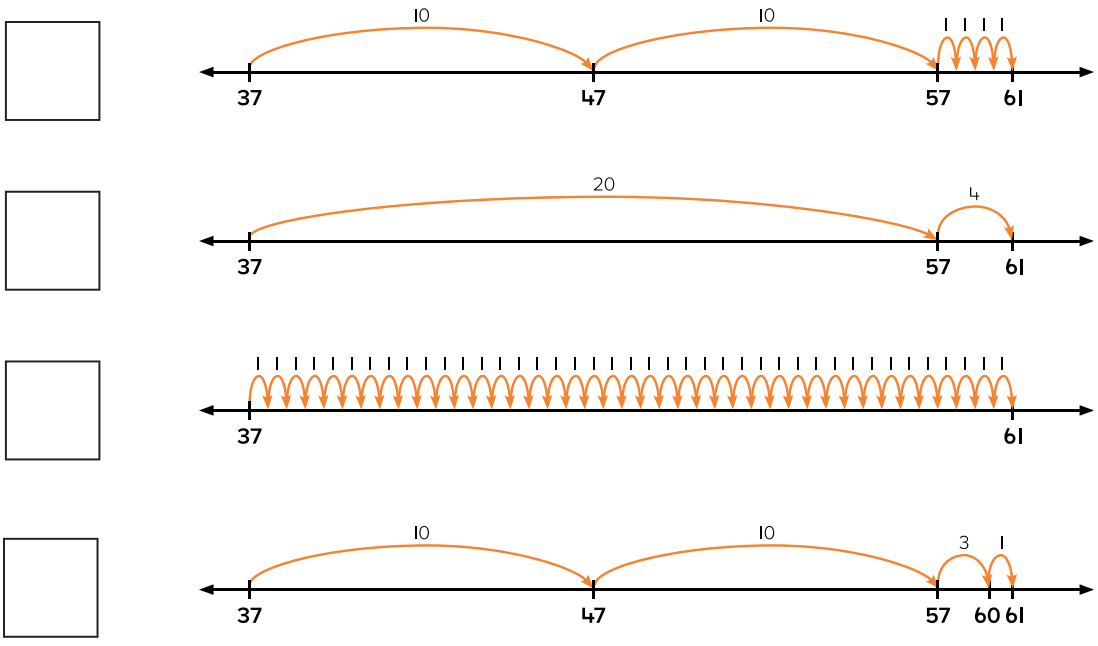
Round 5



65 - 38

Directions : Use the number line below to model the subtraction problem.

Directions : Determine the developmental order.



Directions: Draw a line from each expression to the number line that can be used to model the operation.

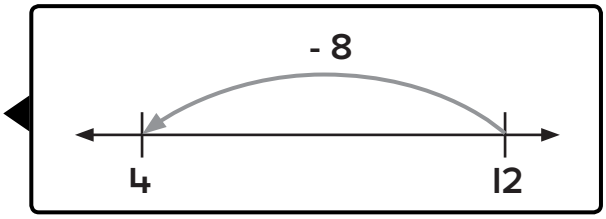
Hint: Some expressions can be matched to more than one number line.

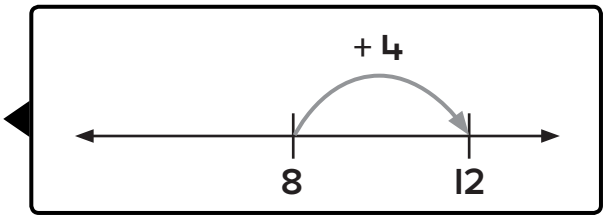
4 + 8

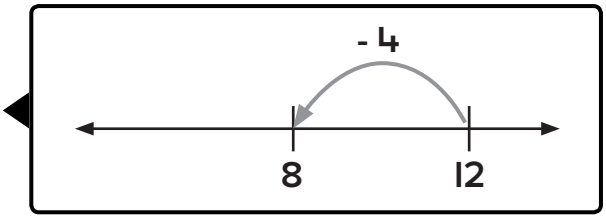
8 + 4

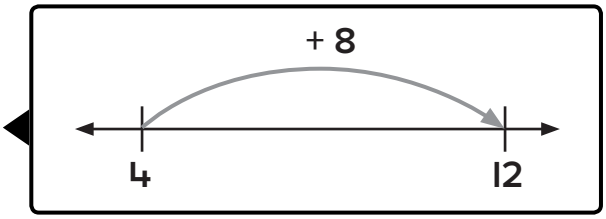
12 - 4

12 - 8









Up or Down

2 or 3 players

Adjusting numbers to subtract

Purpose

In this game, the students are required to find the difference between two two-digit numbers. The students discover that changing both totals in the same way keeps the difference unchanged. This is an efficient mental strategy for subtraction, when the calculation involves bridging across a ten.

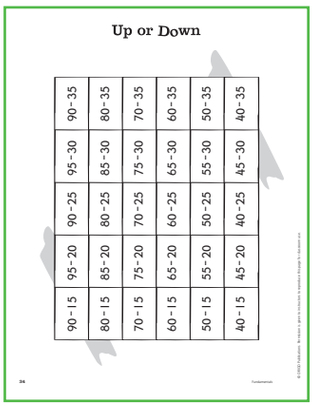
Materials

Each group of players will need

- An 'Up or Down' game board (page 34) as shown below.
- Two (2) number cubes made from blank wooden cubes. One cube should show the numerals 92, 82, 72, 62, 52, and 42. The other cube should show 17, 27, 37, 17, 27, and 37.

Each player will need

- Fifteen (15) counters (a different color for each player).



How to Play

The aim is to arrange three counters adjacently in a horizontal, vertical, or diagonal line.

- The first player rolls the number cubes.
- The player says the subtraction sentence represented by the cubes, then figures out and says an equivalent subtraction sentence.

  Example: Reece rolls 42 and 17, and says, 42 take away 17 is the same as 40 take away 15 or 45 take away 20.

- The player states the difference before claiming a corresponding space on the game board by covering it with a counter. If both choices are unavailable, the player misses a turn.

  Example: Reece says, The answer is 25, and claims 40 – 15 or 45 – 20 on the game board.

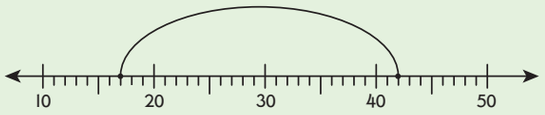
- The other player(s) has a turn.
- The first player to make a line of three adjacent counters is the winner.

Reading the Research

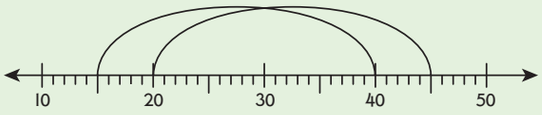
Research shows that children need many different kinds of number experiences to help them deal with real-life situations. It is important that they work regularly with a variety of number representations, including a counting model, a linear model such as a number line, a place-value model and a quantity model (Irons, 2002).

Before the Game

Draw a number line like that shown (right) on an overhead transparency. Invite a volunteer to roll the number cubes and mark the location of the numbers on the line. This number line shows that 42 and 17 was rolled. Place another blank transparency over the number line. Draw a loop to show the difference between the two numbers. Say, ***If we can't quickly calculate the difference, it is sometimes easier to change the two numbers.***



Slide the loop along the number line until the ends are resting on 40 and 15, and ask, ***Have I changed the loop or difference?*** (No.) ***Is it easier to calculate 40 – 15 in our heads?*** (Yes.) Slide the loop in the other direction until the ends are resting on 45 and 20. Again ask, ***Is it easier to calculate 45 – 20?*** (Yes.)



During the Game

Encourage the students to explain the strategy they use to find the answers. Different players will use different strategies. For example, after figuring out that 72 – 37 is the same as 70 – 35, the following players gave these explanations:

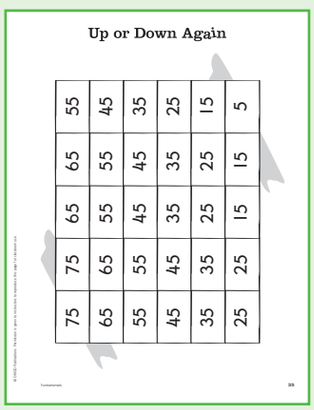
Grace: *I subtracted 30 then another 5. The answer is 35.*

Grant: *I just knew that double 35 is 70, so 70 take away 35 must be 35.*

Gabby: *I started with 35 and added 5 to get 40 and another 30 to get 70. The difference is 5 + 30.*

After the Game

Lead a discussion about the strategy introduced in the game. Ask, ***Did you find this strategy easy (or difficult) to use? Why was it easy (or difficult)? Did you prefer to adjust the numbers down or up? Why?***


Up or Down Again

55	45	35	25	15	5
65	55	45	35	25	15
65	55	45	35	25	15
75	65	55	45	35	25
75	65	55	45	35	25

Beyond the Game

- The students can play 'Up or Down' using a different pair of cubes. On the faces of one cube, write the numerals 18, 28, 38, 18, 28, and 38. The other cube should show 93, 83, 73, 63, 53, and 43.
- The students can play the same game using a different game board. Give them a copy of 'Up or Down Again' shown on page 35 (illustrated). This time, the students verbalize the equivalent number sentence and cover the answer.

Up or Down

90 – 35	80 – 35	70 – 35	60 – 35	50 – 35	40 – 35
95 – 30	85 – 30	75 – 30	65 – 30	55 – 30	45 – 30
90 – 25	80 – 25	70 – 25	60 – 25	50 – 25	40 – 25
95 – 20	85 – 20	75 – 20	65 – 20	55 – 20	45 – 20
90 – 15	80 – 15	70 – 15	60 – 15	50 – 15	40 – 15

Up or Down Again

55	45	35	25	15	5
65	55	45	35	25	15
65	55	45	35	25	15
75	65	55	45	35	25
75	65	55	45	35	25