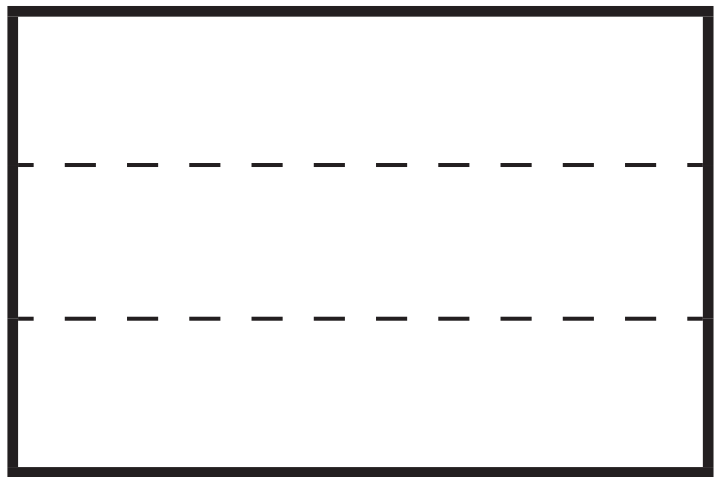
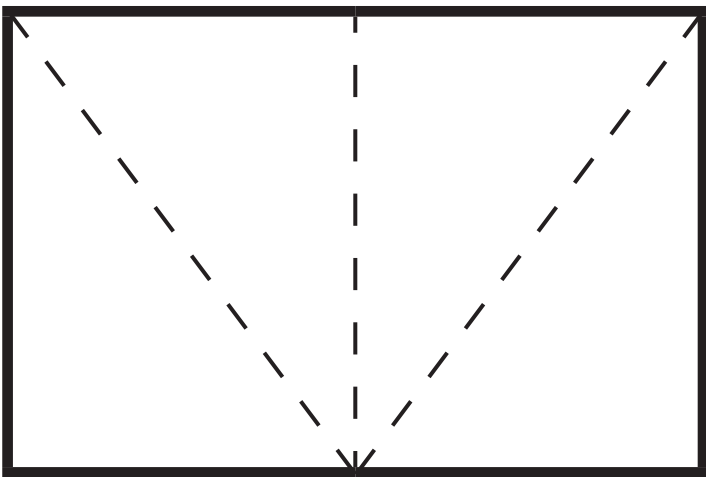
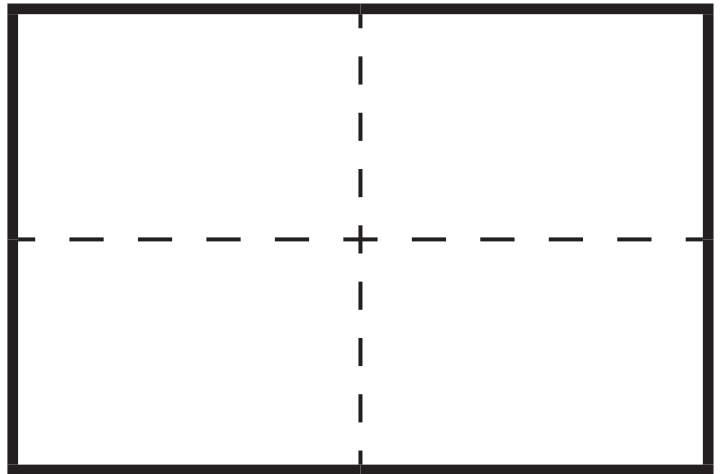
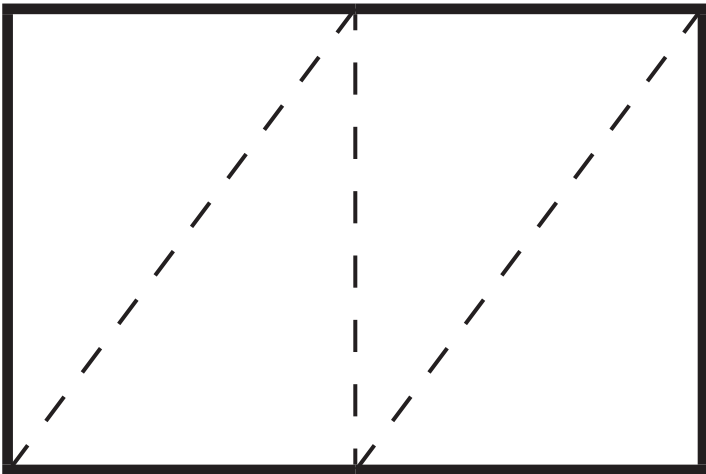
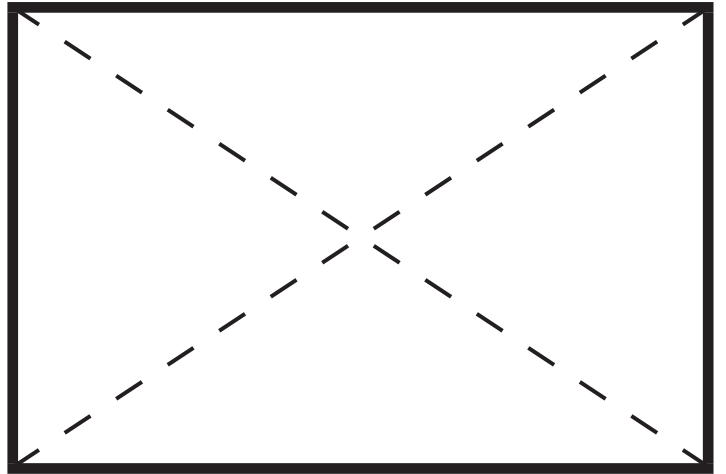
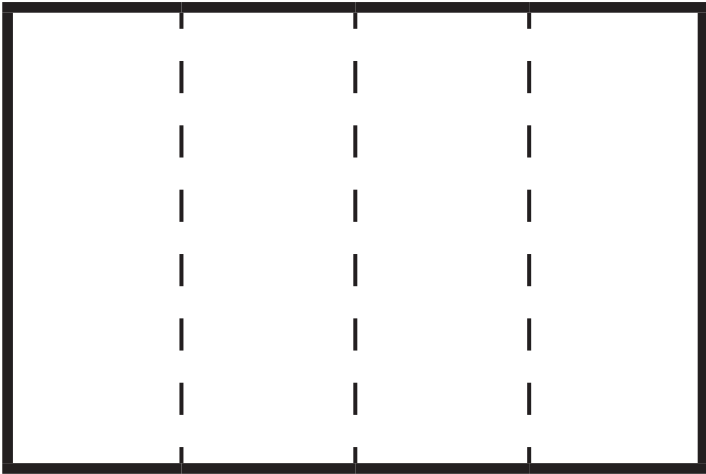




Fractions: From Misunderstanding to Deep Understanding

Debi DePaul

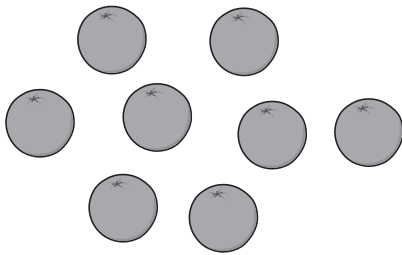


Show $\frac{1}{4}$ in each of these models.

For each model, consider the following questions:

- What is the whole?
- What does equal-sized mean?
- What does the fraction indicate?
- What attribute is the focus?

Set Model:



Area Model:



Length Model:



Number Line Model:



Representing Fractions

Teachers and students need to consider the following:

- The type of quantity (continuous or discrete?) that the model is intended to represent.
- How the whole is defined.
- What equal-sized means in the model.
- What the fraction indicates.



Representing Fractions

Set Model



The number – a discrete (countable) quantity

The whole is determined by a defined **count** of a collection or set.

The **same number** of items represents equal-sized parts.

The fraction indicates the count of objects in the subset compared to the defined set of objects.



Representing Fractions

Area Model



The area – a continuous (measureable) quantity

The whole is determined by the defined **area** or region.

The **same area** represents equal-sized parts.

The fraction indicates the area of the part compared to the area of the whole.



Representing Fractions

Length Model



The length – a continuous (measureable) quantity

The whole is determined by a defined **length**.

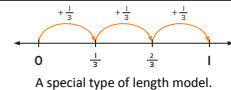
The **same length** represents equal-sized parts.

The fraction indicates the length of the part compared to the length of the whole.



Representing Fractions

Number Line Model



The distance – a continuous (measureable) quantity

The whole is determined by a unit of **distance** from 0 to 1.

The **same distance** represents equal-sized parts.

The fraction indicates the location of a point in relation to the distance from 0 with regard to the defined unit.



Representing Fractions

Type of Model	Type of Quantity	Whole	Meaning of Equal-Sized Parts	What the Fraction Indicates
Set model	discrete	determined by a defined count of a collection or set	same number of items	the count of objects in the subset compared to the defined set of objects
	continuous	determined by a defined area or region	same area	the area of the indicated part compared to the area of the indicated whole
Length model	continuous	determined by a defined length	same length	the length of the indicated part compared to the length of the indicated whole
	continuous	unit of distance from 0 to 1	same distance	the location of a point in relation to the distance from 0 with regard to the defined unit

Suppose that four people are going to share 3 pizzas. If the pizzas are shared equally, how much pizza does each person get?

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Step Up

- I. Complete these to show how you would solve each problem.
Draw lines and color parts to show each share.

- a. Two slices of banana bread are shared equally by 3 people.
How much of one whole slice will be in each share?

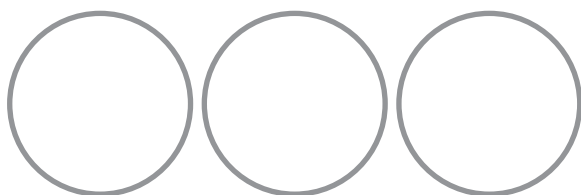


slices shared by

people

$$2 \div \boxed{} = \boxed{}$$

- b. Three pies are shared equally by 8 people. How much of one whole pie will be in each share?



pies shared by

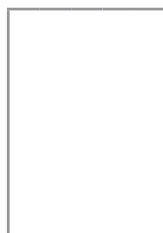
people

$$\boxed{} \div \boxed{} = \boxed{}$$

2. Each large rectangle below is one whole. Complete the sentence, draw lines, and color parts to show each share.

- a. $\frac{3}{5}$ is the same as

divided by



- b. $\frac{4}{7}$ is the same as

divided by

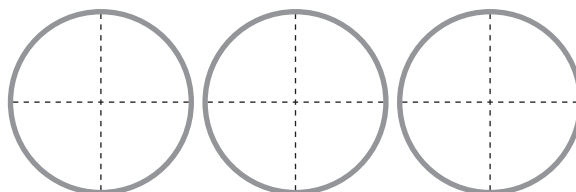


Step Ahead

Complete the sentences so that the quotient in the division sentence is the same as the product in the multiplication sentence. Then use the diagram to show why they are the same.

$$3 \div 4 = \boxed{}$$

$$3 \times \boxed{} = \boxed{}$$



Interpreting Fractions

Fractions as...	Numerator	Denominator	Meaning of $\frac{3}{4}$
Part of a Whole	number of equal-sized parts indicated	number of equal-sized parts in the whole	3 parts out of 4 equal-sized parts
Numbers or Measures	number or count of equal-sized parts (unit fractions)	number of equal-sized parts (unit fractions) needed to create the whole	• 3 counts (repetitions) of the unit fraction $\frac{1}{4}$ • $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$
Quotients	number of items in the whole	number of shares or equal-sized parts	• $3 \div 4$ • the result when divided or shared