CUETHINK

Portrait of a Graduate

White Paper

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Over the past several years, schools across the nation have increasingly taken up the charge to codify the knowledge, skills, and dispositions that all students should have upon graduation. As CueThink's Dr. Laurie Sullivan eloquently detailed in a recent blog post, many of these vision statements—oftentimes referred to as Portraits of a Graduate—promote universal ideologies and aspirations. These universal themes include the desire that all students should become creative and critical thinkers, resilient problem-solvers, cogent and effective communicators, and responsible citizens of our global world.

While few would argue against the importance of these attributes, and while it is easy to see how to integrate one or two of these themes into a given subject, many teachers struggle to integrate all five of them in mathematics classrooms—particularly in light of the demands of high-stakes testing. Fortunately, teaching through problem-solving has not only been shown to increase students' content knowledge, understandings, and performance on standardized tests[1], but also offers a conduit through which teachers can facilitate the development of their students' 21st century skills.

Creative and Critical Thinkers

At its heart, mathematics is about critical thinking and creativity[2]. Mathematical problems can almost always be solved in numerous ways, and when students are given opportunities to think deeply about mathematics they actually become more proficient at mathematical computations as well. For example, the other day in class I asked my students to compare the fractions four-ninths and two-fifths and to justify their answer using a mathematically valid argument. Three of the students' arguments are presented below:

"I knew that four-ninths was half of a ninth less than one half, and two-fifths was a half of a fifth less than one-half. Since ninths are smaller than fifths, half of a ninth is smaller than half of a fifth. Thus, four-ninths is closer to one half than two-fifths, and so four-ninths is larger."

"I know that four-ninths is one-ninth more than three-ninths, and three-ninths is equal to 33.3%. Since the extra one-ninth is one-third of the three-ninths, I need to add one-third of 33.3% to the

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[1] (e.g., Boaler, 2002; Boaler & Staples, 2008; Cai, 2003; Hiebert, 2003)

[2] (Boaler, 2016)

33.3%, which equals 33.3% plus 11.1%, which equals 44.4%. Since two fifths is equal to four-tenths and four-tenths is 40%, four-ninths must be greater than two-fifths.

"I know that two-fifths is equivalent to four-tenths. Since ninths are larger than tenths, four-ninths must be larger than four-tenths."

As these mathematically valid reasonings show, when students are allowed to think creatively about mathematics, they are not only building 21st century skills, but they are also enhancing their abilities to solve basic mathematical computations, which translates into higher test scores and greater access to higher level mathematics courses. However, how do we encourage students to think about mathematics in ways such as this? The answer is by engaging students in problem-solving.

Within classrooms, problem-solving functions as a vehicle to increase the visibility of creativity and critical thinking [3] as students struggle through problems and think deeply about the mathematics. When these ponderings are then coupled with a safe environment that emphasizes reflection, students begin to transfer their creative approaches to mathematics to everything from basic mathematical computations to complex problem-solving scenarios. As they do so, the students begin to see mathematics in a different way, and as several of my students have said, although they never thought of themselves as being "math people," they truly believe that they can excel in mathematics when they are given opportunities to think deeply and to leverage their creative insights into the subject.

Utilizing CueThink further enhances the effectiveness of these aims as students are given opportunities to solve high-quality problems in numerous ways using the myriad different tools that are available within the application. Students and teachers can then watch and discuss the different thinklets that have been created in a way that showcases the creativity inherent in thinking deeply about mathematics. Moreover, the four-phase process scaffolds the problem-solving process in a way that is accessible to all students. Thus, as teachers and students spend more time solving problems within CueThink, they begin to see mathematics as a creative endeavor, rather than an exercise in procedural memorization. Moreover, as students collaborate and discuss mathematical problems, students also learn how to think and reason in a critical manner.

[3] (e.g., Boaler, 2002; Silver, 1997)

Communication and Collaboration

When students collaborate to solve worthwhile problems, they are given opportunities to engage in rich discussions about the mathematics and the problem-solving process. Within this, students are not only required to clearly present and defend their own thinking to others, but they are also required to listen to and understand the thinking of others. Through this process of explaining, listening, questioning, and drawing connections between ideas, students begin to develop holistic understandings of the mathematics, and hone their problem-solving skills as they learn new heuristics. All the while, students are also practicing those important skills of communicating their ideas in a way that others can understand and collaborating with peers to bridge their ideas and then leveraging them in pursuit of a solution.

Within CueThink, these processes are further enhanced as students are encouraged to collaborate with peers to solve problems, and are also required to record their thinking in their problem-solving journals. These journals enhance the learning opportunities for students as they continuously reflect on and write about what they are doing and why they are doing it--thereby maintaining metacognitive awareness. Finally, teachers can bring these mathematical conversations to the full class as they select, show, and discuss students' thinklets and/or allow students to watch and annotate other students' thinklets. All the while, students are refining their collaboration and communication skills, developing deeper understandings of important mathematical ideas, and making connections between the concepts they have learned.

Resilient Problem-Solvers

Problem-solving is, by definition, a challenge for which a student does not immediately know the answer. Thus, the problem-solving process is often complex and amorphous. For this reason, successfully navigating through the labyrinth of problem-solving requires students to be resilient in the face of challenges and adversity. Resilience is largely dependent on students having the tools necessary to be successful, and on students believing that, through hard work, they *will* be successful.

CueThink facilitates the development of both of these aspects of resiliency by providing students with rich mathematical tasks, and the framework through which to engage with them. The CueThink platform explicitly calls students attention to key facets of problem-solving such as sense-making and the use of heuristics, and then scaffolds the process by using Pólya's[4] four phases of problem-solving. Moreover, when students do become frustrated, teachers and/or other students can provide hints through annotations directly within the program to keep the student progressing through the problem without lowering the cognitive demand. By regularly engaging with rich tasks in this way, students develop the skills and beliefs to become proficient problem-solvers in mathematics and beyond.

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4 See Pólya(1945/2014)

Responsible Global Citizens

Finally, mathematical problem-solving can be a conduit through which students engage in global citizenship. From solving problems related to water shortages to analyzing data from polls and elections, mathematical problems are all around us. Engaging students in solving such problems not only offers students windows into the broader world, but also provides students opportunities to engage in productive discourse and debate.

For example, CueThink allows students to operate as critical friends. Operating within this capacity, students actively engage with the ideas of others and then politely critique and push back on ideas when necessary. This process is a staple of our democracy and led Goodman, Sands, and Coley[5] to argue that problem-solving proficiency is essential to ensure that citizens can engage in “meaningful participation in our democratic institutions” (p. 5). Thus, as students utilize the annotations feature of CueThink, they practice listening to and critiquing the ideas of others in a productive and meaningful manner that moves the conversation forward rather than simply attacking the person. In this way, students become more ethical global and digital citizens with the skills necessary to tackle the global and political problems of the 21st century.

Overall, regularly incorporating problem-solving into mathematics offers a gateway for teachers to meaningfully integrate 21st century skills—such as those captured by the Profile of a Graduate movement—into mathematics classrooms. Creative and critical thinking, collaboration and communication, resilient problem-solving, and global citizenship are not afterthoughts to teaching, but are instead the byproduct of good teaching.

[5] See Goodman, Sands, and Coley (2015)

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