


Develop Mathematical Practices with Computational Thinking

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Chair, Mathematics Advisory Board
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Computational thinking refers to the thought processes involved in expressing solutions as computational steps or algorithms that can be carried out by a computer.

CSTA Standards, citing
Cuny, Snyder, & Wing, 2010; Aho, 2011; Lee, 2016

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"If I'm using tech in any way in my job, it's incumbent that we all speak a common language, and that's really the language of **computational thinking**."

Richard Culatta
Chief Executive Officer, ISTE


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ISTE Computational Thinking Competencies for Educators

- Educators continually improve their practice by developing an understanding of computational thinking and its application as a cross-curricular skill. Educators develop a working knowledge of core components of computational thinking....
- Educators facilitate learning by integrating computational thinking practices into the classroom. Since computational thinking is a foundational skill, educators develop every student's ability to recognize opportunities to apply computational thinking in their environment.


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Computer Science Practices

- P3: Recognizing and defining computational problems.
 - P3.2. **Decompose complex real-world problems** into manageable subproblems that could integrate existing solutions or procedures
- P4: Developing and using abstractions.
 - P4. 1. **Extract common features** from a set of interrelated processes or complex phenomena.
- P6: Testing and refining computational artifacts.
 - P6.2. **Identify and fix errors** using a systematic process.

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BUT IS THIS IDEA IN OTHER STANDARDS?

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NGSS Science and Engineering Practices

- Practice 5 – Use mathematics and **computational thinking**.






BUT IS COMPUTATIONAL THINKING IN OUR MATH STANDARDS?






Standards for Mathematical Practice

- SMP 2 – Reason abstractly and quantitatively.
- SMP 4 – Model with mathematics.
- SMP 7 – Look for and make use of structure.
- SMP 8 – Look for and express regularity in repeated reasoning.

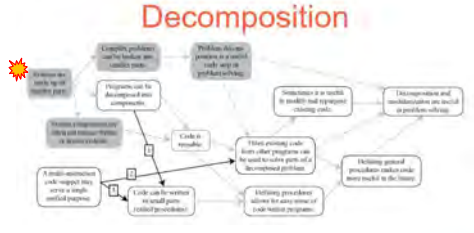



WHAT ABOUT LEARNING TRAJECTORIES?

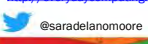




Learning Trajectories for Computational Thinking



Decomposition

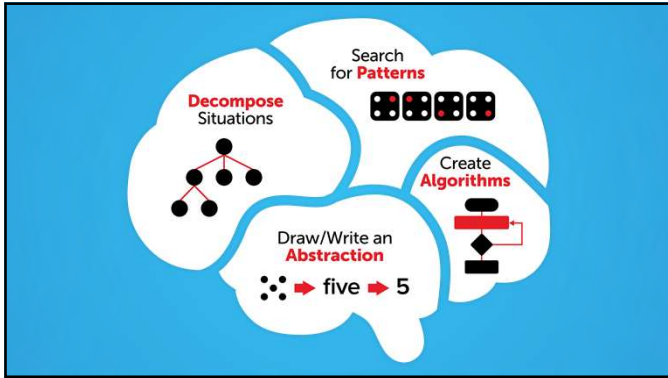


<http://everydaycomputing.org/public/visualization/>


ELEMENTS OF COMPUTATIONAL THINKING



Decompose Situations


- Breaking down a complex problem into manageable parts.



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Search for Patterns


- Looking for similarities among and within problems, including identifying a recurring sequence or structure.



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Draw/Write Abstractions


- Identifying the key or useful elements and ignore what is irrelevant to represent, describe, or summarize the information.



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Create Algorithms

- Developing a step-by-step sequence of rules to solve a problem.



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WHAT DOES THIS LOOK LIKE IN MATH CLASS?

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Simple Steps

- How many purple squares are in
 - Picture 1?
 - Picture 2?
 - Picture 3?
 - Picture 5?
 - Picture 10?
- How many yellow squares are in
 - Picture 1?
 - Picture 2?
 - Picture 3?
 - Picture 5?
 - Picture 10?

Algebra for All, orange level
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Where's the computational thinking?

- Decomposition
 - focus on one color tile; see the parts separately (SMP 7 – structure)
- Patterns
 - how does the number of tiles change with each step? (SMP 7 – structure)
- Abstractions
 - can we represent step 5 or step 10 without building them all? (SMP 2 – decontextualize)
- Algorithms
 - can we describe the pattern so we can make predictions? (SMP 4 – modeling & SMP 8 – repeated reasoning)

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Let's do some math

- Build the odd numbers.

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Let's do some math

- Build the odd numbers.

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Let's do some math

- Build the odd numbers.
- What patterns do you see?
- How many cubes are in the 10th term?
- How many cubes are in the nth term?

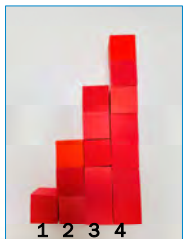
Pause & Build

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Building odd numbers

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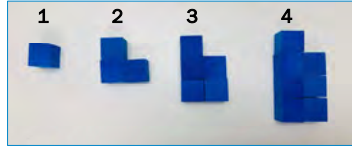
What's the ____ term?



What is the 10th term?
 How do you know?

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What's the ____ term?

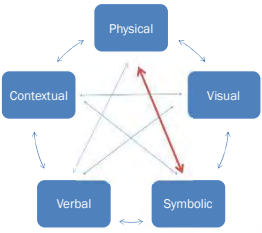


What is the 10th term?
 How do you know?

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How do our representations of the patterns become more abstract?

- Contextualize and decontextualize
- See the problem through a mathematical lens

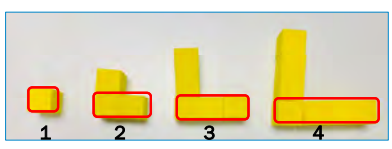


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DOES THE REPRESENTATION CHANGE YOUR THINKING ABOUT FUTURE TERMS?

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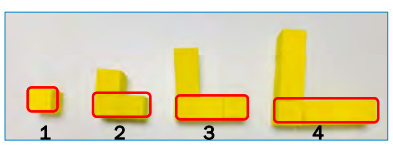
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What is the 10th term?
 What is the nth term?
 How do you know?

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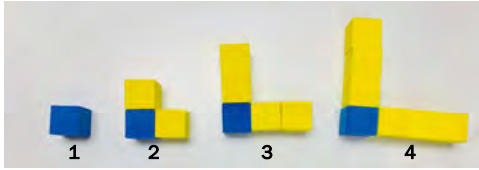
What's the nth term?




$n + (n - 1)$

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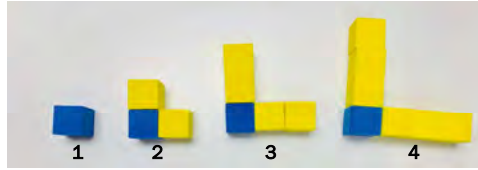
What's the nth term?




Decomposition: What are the parts of this structure?

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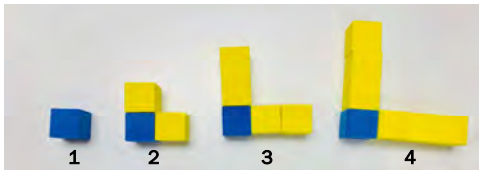
What's the nth term?




Decomposition: What are the parts of this structure?
 Algorithmic Thinking: How do you describe the growing pattern in words?

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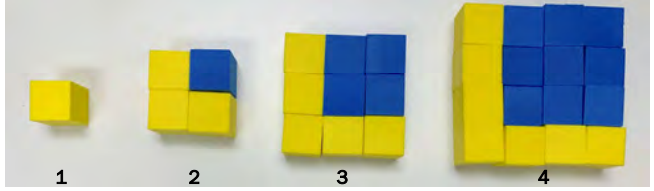
What's the nth term?




Algorithmic Thinking (symbols): $1 + 2(n-1)$

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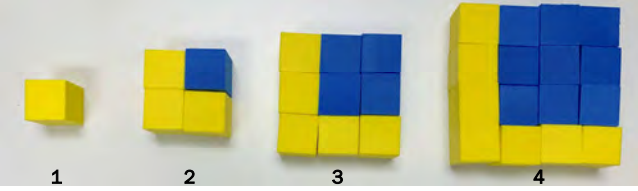
What's the nth term?




Decomposition: What are the parts of this structure?

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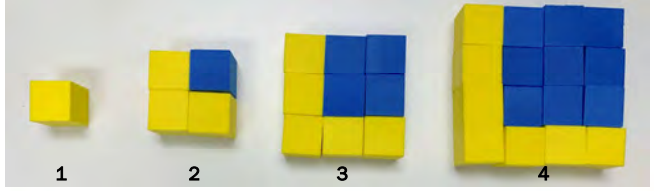
What's the nth term?




Decomposition: What are the parts of this structure?
 Algorithmic Thinking: How do you describe the growing pattern in words?

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What's the nth term?



Algorithmic Thinking (symbols): $n^2 - (n - 1)^2$

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Another potential pattern

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Frieze Frame

If the frieze has 12 hexagons, how many triangles does it have?
 If the frieze has 46 triangles, how many hexagons does it have?
 How do you know?

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Decomposing Frieze Frame

Term 1 Term 2 Term 3

What is there to start?
 What is added for each term?

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Decomposing Frieze Frame

Term 1 Term 2 Term 3

What will change for term 4?

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Thinking Algorithmically: Frieze Frame

Term 1 Term 2 Term 3

H 2T + H

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Thinking Algorithmically: Frieze Frame


Term 1 Term 2 Term 3


What do we start with? **One yellow (H)**
 What do we add with each term? **Two greens & 1 yellow (2T + H)**


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**Thinking Algorithmically:
Frieze Frame**


Term 1  H


Term 2  $H + (2T + H)$


Term 3  $H + (2T + H) + (2T + H)$


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**Thinking Algorithmically:
Frieze Frame**


Term 1  H


Term 2  $H + (2T + H)$


Term 3  $H + (2T + H) + (2T + H)$


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**Thinking Algorithmically:
Frieze Frame**


Term 1  H


Term 2  $H + (2T + H)$


Term 3  $H + (2T + H) + (2T + H)$


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**Thinking Algorithmically:
Frieze Frame**


Term 1  H


Term 2  $H + (2T + H)$


Term 3  $H + (2T + H) + (2T + H)$
 $= H + 2(2T + H)$


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**Thinking Algorithmically:
Frieze Frame**

Term 1  H


Term 2  $H + (2T + H)$

Term 3  $H + (2T + H) + (2T + H)$
 $= H + 2(2T + H)$
 $= H + (n-1)(2T + H)$

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Progression of Algebraic Thinking

Letters as Descriptions/Labels	Letters as Variables
<ul style="list-style-type: none"> • Shorthand for what we see or what happens • Fairly literal representations • Know/write the words 	<ul style="list-style-type: none"> • Values actually vary • Other values depend on that variation • Reflect more formal mathematical relationships

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Progression of Algebraic Thinking

Letters as Descriptions/Labels

- Shorthand for what we see or what happens
- Fairly literal representations
- Know/write the words

Number of Triangles
 Term 3: 2 sets of 2T
 Term n: ? sets of 2T

Letters as Variables

- Values actually vary
- Other values depend on that variation
- Reflect more formal mathematical relationships

Number of Triangles
 Term 3: $2(2) = 4$
 Term n: $2(n - 1)$

Remember the students/professors problem?

- There are 6 students for each professor at the university.
- Which is the best equation to represent this situation?

Six times the number of professors is the number of students

$6s = 1p$

"Six students correspond to one professor"

$6p = s$

Frieze Frame: Let's Test It!

$H + (n-1)(2T + H)$

Term 1

Term 2

Term 3

Term	Hexagons	Triangles
1	1	0
2	2	2
3	3	4
4		
n		

Frieze Frame: Let's Test It!

$H + (n-1)(2T + H)$

Term 1

Term 2

Term 3

Term	Hexagons	Triangles
1	1	0
2	2	2
3	3	4
4	4	6
n		

Frieze Frame: Let's Test It!

$H + (n-1)(2T + H)$

Term 1

Term 2

Term 3

Term	Hexagons	Triangles
1	1	0
2	2	2
3	3	4
4	4	6
n	n	$2(n-1)$

Frieze Frame: Let's Test It!

$H + (n-1)(2T + H)$

Term 1

Term 2

Term 3

Term	Hexagons	Triangles
1	1	0
2	2	2
3	3	4
4	4	6
n	n	$2(n-1)$

If the frieze has 12 hexagons, how many triangles does it have?

Frieze Frame: Let's Test It!

$$H + (n-1)(2T + H)$$

Term	Hexagons	Triangles
1	1	0
2	2	2
3	3	4
4	4	6
n	n	2(n-1)

If the frieze has 46 triangles, how many hexagons does it have?

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How have we....

- Identified patterns?
- Decomposed these patterns?
- Worked abstractly?
- Developed algorithms?

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Standards for Mathematical Practice

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Additional Resources

- Books with patterns also support computational thinking
 - *Patterns Here, Patterns There*
 - *Perfect Patterns*
- Handout uploaded at NCTM
- Digital big books and lesson plans for Simple Steps and Frieze Frame online at ORIGO

<https://www.origoeducation.com/nctm-feb21>

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