Develop Mathematical Practices with Computational Thinking

Sara Delano Moore, Ph.D.
Director of Professional Learning
Chair, Mathematics Advisory Board
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Computational thinking refers to the thought processes involved in expressing solutions as computational steps or algorithms that can be carried out by a computer.

CSTA Standards, citing
Cuny, Snyder, & Wing, 2010; Aho, 2011; Lee, 2016

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"If I'm using tech in any way in my job, it's incumbent that we all speak a common language, and that's really the language of **computational thinking.**"

Richard Culatta
Chief Executive Officer, ISTE

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ISTE Computational Thinking Competencies for Educators

- Educators continually improve their practice by developing an understanding of computational thinking and its application as a cross-curricular skill. Educators develop a working knowledge of core components of computational thinking....
- Educators facilitate learning by integrating computational thinking practices into the classroom. Since computational thinking is a foundational skill, educators develop every student's ability to recognize opportunities to apply computational thinking in their environment.

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Computer Science Practices

- P3: Recognizing and defining computational problems.
  - P3.2. **Decompose complex real-world problems** into manageable subproblems that could integrate existing solutions or procedures
- P4: Developing and using abstractions.
  - P4. 1. **Extract common features** from a set of interrelated processes or complex phenomena.
- P6: Testing and refining computational artifacts.
  - P6.2. **Identify and fix errors** using a systematic process.

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BUT IS THIS IDEA IN OTHER STANDARDS?

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NGSS Science and Engineering Practices

- Practice 5 – Use mathematics and computational thinking.

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BUT IS COMPUTATIONAL THINKING IN OUR MATH STANDARDS?

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Standards for Mathematical Practice

- SMP 2 – Reason abstractly and quantitatively.
- SMP 4 – Model with mathematics.
- SMP 7 – Look for and make use of structure.
- SMP 8 – Look for and express regularity in repeated reasoning.

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WHAT ABOUT LEARNING TRAJECTORIES?

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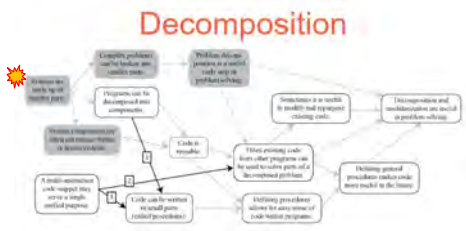
Learning Trajectories for Computational Thinking



http://everydaycomputing.org/public/visualization/

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ELEMENTS OF COMPUTATIONAL THINKING

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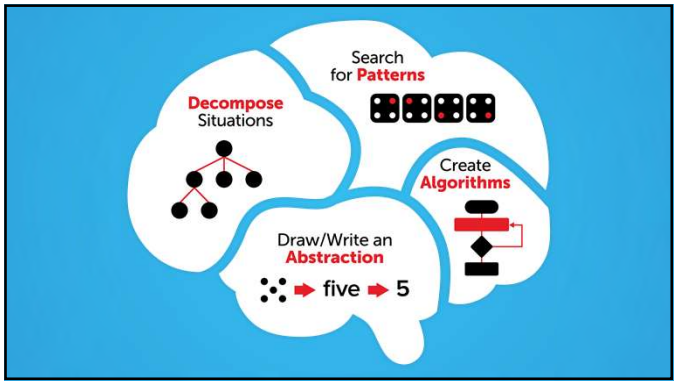

Decompose Situations

- Breaking down a complex problem into manageable parts.

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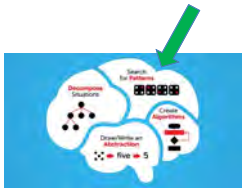
Search for Patterns

- Looking for similarities among and within problems, including identifying a recurring sequence or structure.



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Draw/Write Abstractions

- Identifying the key or useful elements and ignore what is irrelevant to represent, describe, or summarize the information.

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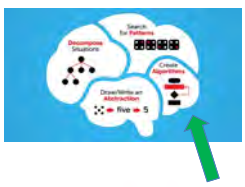
Create Algorithms

- Developing a step-by-step sequence of rules to solve a problem.



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WHAT DOES THIS LOOK LIKE IN MATH CLASS?

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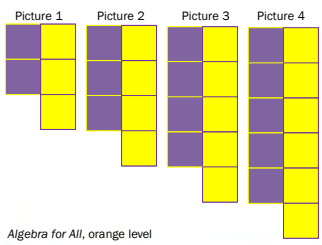
Simple Steps



Algebra for All, orange level

- How many purple squares are in
  - Picture 1?
  - Picture 2?
  - Picture 3?
  - Picture 5?
  - Picture 10?
- How many yellow squares are in
  - Picture 1?
  - Picture 2?
  - Picture 3?
  - Picture 5?
  - Picture 10?

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Where's the computational thinking?

- Decomposition
  - focus on one color tile; see the parts separately (SMP 7 – structure)
- Patterns
  - how does the number of tiles change with each step? (SMP 7 – structure)
- Abstractions
  - can we represent step 5 or step 10 without building them all? (SMP 2 – decontextualize)
- Algorithms
  - can we describe the pattern so we can make predictions? (SMP 4 – modeling & SMP 8 – repeated reasoning)

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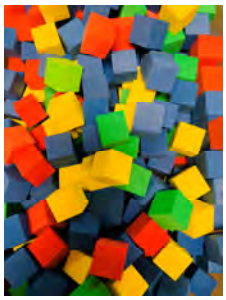
Let's do some math

- Build the odd numbers.



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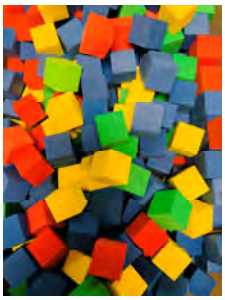
Let's do some math

- Build the odd numbers.



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Let's do some math

- Build the odd numbers.
- What patterns do you see?
- How many cubes are in the 10th term?
- How many cubes are in the nth term?

Pause & Build

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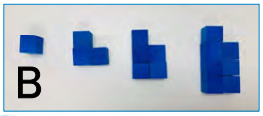
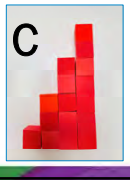
Building odd numbers





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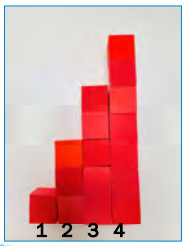
What's the _____ term?



What is the 10th term?
How do you know?

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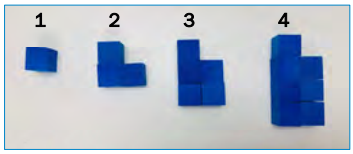
What's the _____ term?



What is the 10th term?
How do you know?

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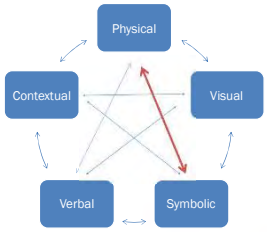
How do our representations of the patterns become more abstract?

- Contextualize and decontextualize
- See the problem through a mathematical lens



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DOES THE REPRESENTATION CHANGE YOUR THINKING ABOUT FUTURE TERMS?

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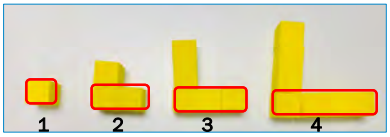
What's the _____ term?



What is the 10th term?
What is the nth term?
How do you know?

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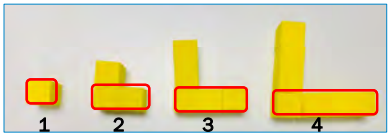
What's the nth term?



n + (n – 1)

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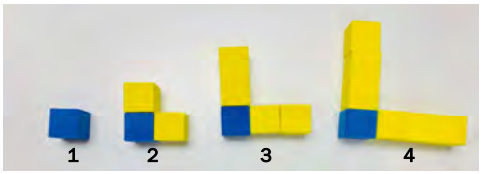
What's the n^{th} term?



Decomposition: What are the parts of this structure?

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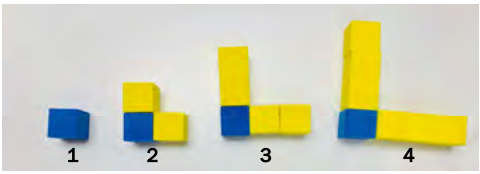
What's the n^{th} term?



Decomposition: What are the parts of this structure?
Algorithmic Thinking: How do you describe the growing pattern in words?

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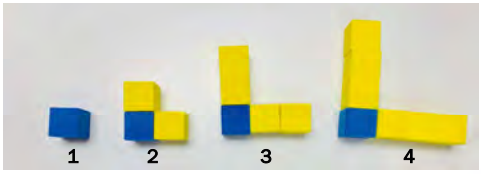
What's the n^{th} term?



Algorithmic Thinking (symbols): $1 + 2(n-1)$

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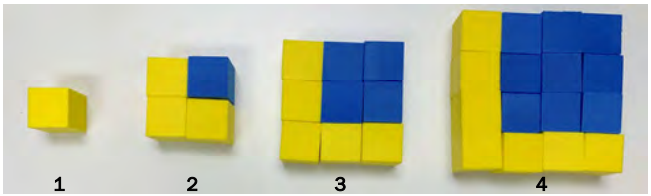
What's the n^{th} term?



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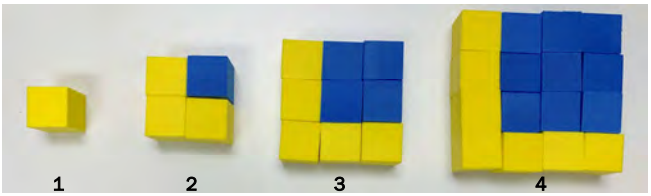
What's the n^{th} term?



Decomposition: What are the parts of this structure?
Algorithmic Thinking: How do you describe the growing pattern in words?

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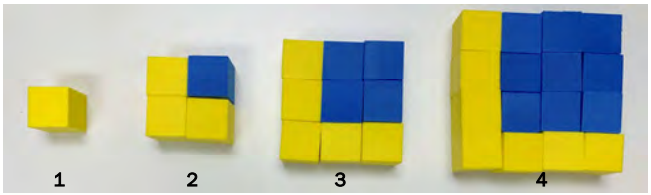
What's the n^{th} term?



Algorithmic Thinking (symbols): $n^2 - (n - 1)^2$

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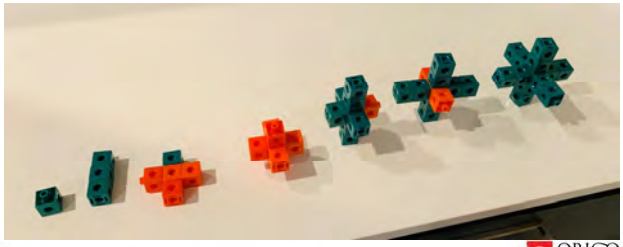
Another potential pattern



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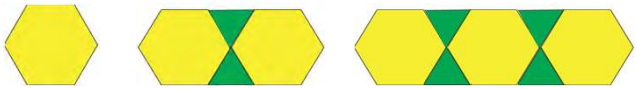
Frieze Frame



If the frieze has 12 hexagons, how many triangles does it have?

If the frieze has 46 triangles, how many hexagons does it have?

How do you know?

Algebra for All, red level
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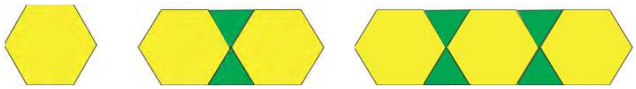
Decomposing Frieze Frame



Term 1 Term 2 Term 3

What is there to start?
What is added for each term?

Algebra for All, red level
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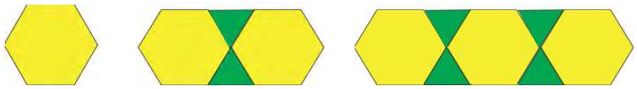
Decomposing Frieze Frame



Term 1 Term 2 Term 3

What will change for term 4?

Algebra for All, red level
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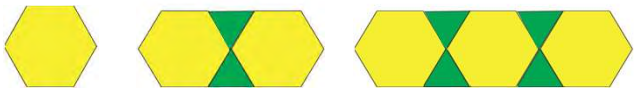
Thinking Algorithmically: Frieze Frame



Term 1 Term 2 Term 3

H

2T + H

Algebra for All, red level
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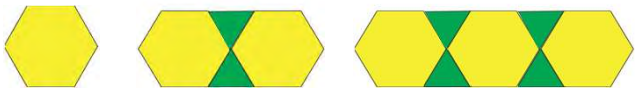
Thinking Algorithmically: Frieze Frame



Term 1 Term 2 Term 3

What do we start with? One yellow (H)
What do we add with each term? Two greens & 1 yellow (2T + H)

Algebra for All, red level
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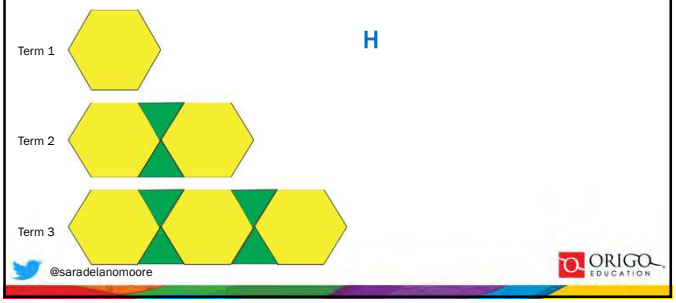
Thinking Algorithmically: Frieze Frame



H

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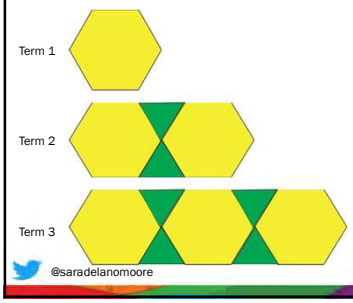
Thinking Algorithmically: Frieze Frame



H

H + (2T + H)

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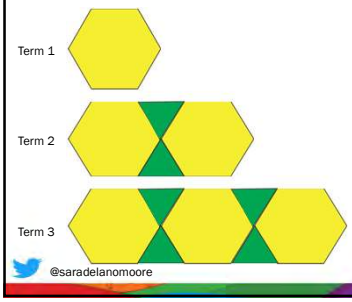
Thinking Algorithmically: Frieze Frame



H

H + (2T + H)

H + (2T + H) + (2T + H)

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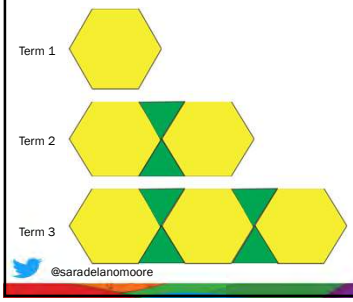
Thinking Algorithmically: Frieze Frame



H

H + (2T + H)

H + (2T + H) + (2T + H)
= H + 2(2T + H)

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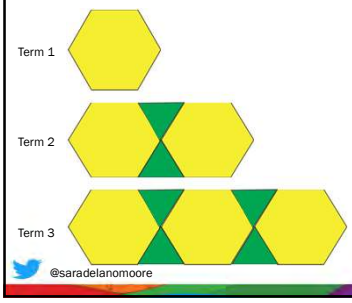
Thinking Algorithmically: Frieze Frame



H

H + (2T + H)

H + (2T + H) + (2T + H)
= H + 2(2T + H)
= H + (n-1)(2T + H)

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Progression of Algebraic Thinking

Letters as Descriptions/Labels

- Shorthand for what we see or what happens
- Fairly literal representations
- Know/write the words

Letters as Variables

- Values actually vary
- Other values depend on that variation
- Reflect more formal mathematical relationships

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Progression of Algebraic Thinking

Letters as Descriptions/Labels
- Shorthand for what we see or what happens
- Fairly literal representations
- Know/write the words

Number of Triangles
Term 3: 2 sets of 2T
Term n: ? sets of 2T

Letters as Variables
- Values actually vary
- Other values depend on that variation
- Reflect more formal mathematical relationships

Number of Triangles
Term 3: 2(2) = 4
Term n: 2(*n* – 1)

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Remember the students/professors problem?

- There are 6 students for each professor at the university.
- Which is the best equation to represent this situation?

Six times the number of professors is the number of students

$6p = s$

$6S = 1P$

"Six students correspond to one professor"

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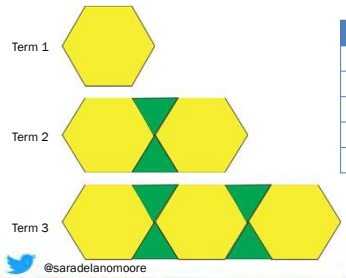
Frieze Frame: Let's Test It!

H + (n-1)(2T + H)



Term	Hexagons	Triangles
1	1	0
2	2	2
3	3	4
4		
n		

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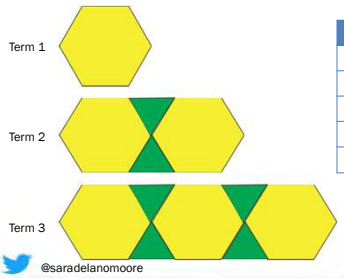
Frieze Frame: Let's Test It!

H + (n-1)(2T + H)



Term	Hexagons	Triangles
1	1	0
2	2	2
3	3	4
4	4	6
n		

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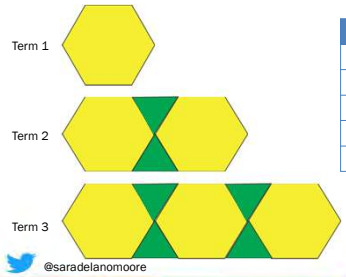
Frieze Frame: Let's Test It!

H + (n-1)(2T + H)



Term	Hexagons	Triangles
1	1	0
2	2	2
3	3	4
4	4	6
n	n	2(n-1)

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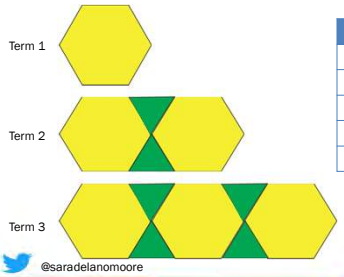
Frieze Frame: Let's Test It!

H + (n-1)(2T + H)



Term	Hexagons	Triangles
1	1	0
2	2	2
3	3	4
4	4	6
n	n	2(n-1)

If the frieze has 12 hexagons, how many triangles does it have?

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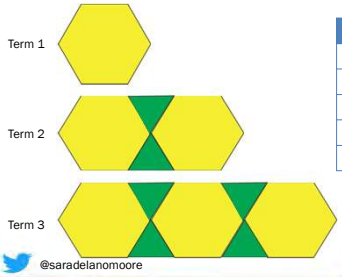
Frieze Frame: Let's Test It!

$H + (n-1)(2T + H)$



Term	Hexagons	Triangles
1	1	0
2	2	2
3	3	4
4	4	6
n	n	2(n-1)

If the frieze has 46 triangles, how many hexagons does it have?

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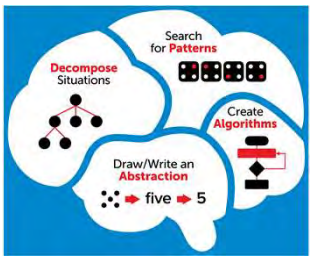
How have we....

- Identified patterns?
- Decomposed these patterns?
- Worked abstractly?
- Developed algorithms?



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Standards for Mathematical Practice

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- SMP 7 – Look for and make use of structure.
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Richard Culatta
Chief Executive Officer, ISTE

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Additional Resources

- Handout uploaded at NCTM
- Digital big books and lesson plans for Simple Steps and Frieze Frame online at ORIGO

  https://www.origoeducation.com/nctm-apr21

- Books with patterns also support computational thinking
  - *Patterns Here, Patterns There*
  - *Perfect Patterns*

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