What a Concept: Using Purposeful Talk to Foster Understanding of Mathematical Concepts

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If You Build It, They Will Think and Talk & Think...

First Build

- The task and the timing of tasks
- The groups
- The physical workspace
- Facilitation & feedback skills

Other Considerations

- Homework
- Note-taking
- Evaluation and grading

Liljedahl, P. (2021). *Building thinking classrooms in mathematics grades K-12: 14 teaching practices for enhancing learning*. Corwin: Thousand Oaks, CA. p. 11.

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Within the First Five Minutes

- Start with non-curricular, high engaging tasks
- Shift to scripted curriculum thinking tasks
- Script your curricular tasks
  Ask a prior knowledge question
  Ask an extension question
  Ask students to do something without telling them how

Engage students in discourse that includes

- Articulation of mathematical ideas
- Connecting representations and abstractions
- Productive lingering

Let go of assumptions that:

- Students understand operations because they have performed computation
- Students no longer need to dig into operations after working with an operation for a number of years
- All students understand an operation once one student states it
- Students have a deep understanding of all the ideas they articulate

Questions for productive lingering

- What do you notice? Are their patterns or regularities?
- Can you describe what you notice? Can you make a conjecture?
- How can you use a representation to explain? How does your representation support your conjecture? Can you use it to make a generalization?

High Level Discourse

Teacher as facilitator, guide on the side

Student initiated talk, including questions directed to each other

Teacher guides students to contrast strategies

Students justify own thinking

Students use math drawings to describe their thinking and the thinking of other students

Students support and shape each other's thinking

Discourse and Metacognition – Teacher Questions

Tell your partner what you think the problem is about.

Make a list of the things you understand about the problem.

Make a list of the things you understand about the problem?

Self-questioning

- What do I know about the problem?
- What is the problem asking me to find out?
- What strategies can I use to understand the problem better?
- Have I seen something like this before?

Self-reflection

- How is my answer similar to/different from my other students' solutions?
- How do I know my solution is correct?
- How well did I communicate my thinking?
- Could I have done this a different way?
- What if…?

Support Discourse with Talk Moves and Sentence Stems

Clarification and explanation

- Could you describe what you mean?

Justification

- How did you know?

Recognize and challenge misconception

- I don't agree because..
- Have you considered an alternative?

Interpret and use other's statements

- I heard Charla say…and that makes me think…

Require evidence

- Can you give me an example?

Additional Questions to Support Discourse

What decisions did you make?

Can you tell me more about…?

Can you explain a different way?

What patterns do you notice?

How does ____ relate to ____?

What can you tell me without solving the problem (performing computation)?

Rather than trying to add the numbers, try thinking about the pattern.

What do you think about Jorge's question/statement?

Who can repeat what Jessica said in their own words?

The Influence of Tasks on Discourse

1. Does the problem involve meaningful mathematics?
2. Does the problem provide an opportunity for students to apply and extend mathematics?
3. Is the problem interesting to students?
4. Is the problem challenging for students?
5. Does the problem support the use of multiple strategies?
6. Will students' interactions with the problem reveal information about students' mathematical understanding?

Surface Learning

Initiation to new ideas

Begins with development of conceptual understanding

Followed by associated procedural skills

What did you notice?

How does this connect to our model?

What would happen next?

What is this called?

How can I write this?

What does this symbol represent?

Deep Learning

Consolidating understanding of concepts and procedures

Making connections among ideas

Did you notice any patterns that helped you determine where to place your counter and score more points? If so, what were those patterns?

Did your partner have a different strategy? How was it different? Can you think of any ways to improve your strategy or your partner's strategy?

Utilize the questions from talk moves.

You are going to play a game in pairs. Thinking about patterns may be helpful.

The cube has the numbers from 3-8. We are going to multiply the number we roll by 8.

Before we roll, we have to predict the product, verbalize the strategy we are using, and place our counter on the number we predicted.

Use tally marks to tally your score, based on our scoring system.

The first person to reach 30 points wins.

Product matches counter – 5 points

Product in same column as counter – 2 points

Product in same row as counter – 1 point

21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70

Directions

- Predict the product and place your counter on the number.
- Take turns rolling the number cube and using the double-double-double strategy to multiply the number by 8.
- Record the number of points each person received for the round.
- The first person who reaches 30 or more points wins.

Points

- 5 points – The product matches the counter.
- 2 points – The product is in the same column as the counter.
- 1 point – The product is in the same row as the counter.

Transfer Learning

Apply learning to new situations

Think metacognitively.

- Self-questioning
- Self-reflection

Written discourse

Near and far transfer

Discourse and Metacognition – Teacher Questions

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Task Examples

Two friends are playing a game. They each pick up a card and then compare the decimal fraction they chose. The decimal fraction on Sheree's card has two decimal places. The decimal fraction on Oscar's card has only one decimal place. Oscar thinks he holds the greater decimal fraction. Is this possible?

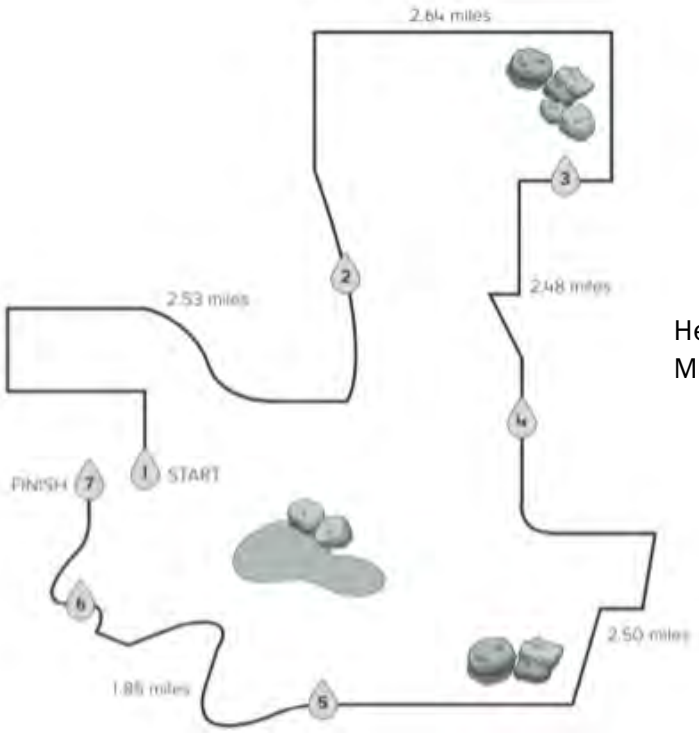
ORIGO Stepping Stones • Grade 5

Andrew has fewer than 60 cubes. He split them into 3 equal groups. How many cubes could be in each group and how many cubes could Andrew have in total?

ORIGO Stepping Stones • Grade 3

Grade 5 Thinking Task

- The Grade 5 Leadership Team has volunteered to help at their city's Annual Marathon.
- They will prepare coolers of energy drinks for the water stops on the 13.1 mile Half Marathon course and sell homemade energy bars at the finish line.



Here is the Course Map for the Half Marathon with the water stops marked.

Apple Pie Granola Bar Recipe

1 batch: 20 bars

Prep time: 25 minutes
Cook time: 10 minutes

Ingredients

2 cups oats

$\frac{1}{2}$ cup shredded, unsweetened coconut

$\frac{1}{2}$ cup pecans, chopped

$\frac{1}{3}$ cup dates

$\frac{1}{4}$ cup coconut oil

1 teaspoon vanilla

$\frac{1}{3}$ cup honey

1 tablespoon cinnamon

$\frac{1}{2}$ teaspoon nutmeg

1 cup dried apples, chopped

Fig and Walnut Bars Recipe

1 batch: 12 two-inch bars

Prep time: 20 minutes
Cook time: 40 minutes

Ingredients

1 $\frac{1}{2}$ cups raw walnuts, chopped

$\frac{1}{3}$ cup whole-grain flour

$\frac{1}{8}$ teaspoon baking soda

$\frac{1}{8}$ teaspoon baking powder

$\frac{1}{4}$ teaspoon sea salt

$\frac{1}{2}$ teaspoon cinnamon

$\frac{1}{3}$ cup light brown sugar

$\frac{1}{2}$ cup dried cranberries

1 cup dried Turkish figs, quartered

1 cup dried apricots, chopped

1 large egg

1 teaspoon pure vanilla extract

Use the **Course Map** to solve.

1. There was a problem with the printer and the course maps were printed without the distance between water stops 6 and 7. How far is the last section of the Half Marathon? Show your thinking.

Use the **Course Map** to solve.

2. The Marathon organizers will purchase energy powder. The Leadership Team will prepare a 10 gallon cooler of energy drink for each water stop. One three-pound tub of energy drink powder makes 24 quarts and costs $8.50. How many tubs will they have to buy and how much will it cost?

   To answer this question, include:

   - The number of energy drink tubs they will need to buy.
   - How much all of the tubs will cost.

3. Many of the ingredients for both the 16 batches of **Apple Pie Granola Bars** and the 20 batches of **Fig Walnut Energy Bars** are donated by the PTA, **except** for the nuts and dried fruits. Families contribute what they have at home. Students make a list of what was donated.

   How much more will they have to buy to make the multiple batches of energy bars?

   For this question:

   - Fill in the **Ingredients Needed** column of the table for all of the ingredients listed.

Recipe	Ingredient Donated	Quantity Donated (in cups)	Ingredients Needed (in cups)
Fig Walnut Energy Bar	Walnuts	10 cups	
	Cranberries	$5\frac{1}{2}$ cups	
	Figs	$7\frac{3}{4}$ cups	
	Apricots	9 cups	
Apple Pie Granola Bar	Coconut	$2\frac{1}{4}$ cups	
	Pecans	$5\frac{3}{4}$ cups	
	Dates	$3\frac{1}{3}$ cups	
	Dried Apples	$12\frac{1}{4}$ cups	

Use the information from the Recipes and Question 4 to solve.

After the Marathon ¼ of the Apple Pie Granola Bars and the Fig and Walnut Bars were left. Students calculated how many bars were sold and how much money they made. They charged $0.50 for each bar.

Cody predicted that after buying the fruits and nuts, they lost money and should have charged $1.00 for each bar.

Nancy argued that they did make money and might have sold fewer bars if they were more than $0.50.

Do you agree with Cody or Nancy? Write a letter to the Leadership Team explaining why.

Include this information in your letter:

- Who do you agree with and why?
- How many bars were sold, how much money was made, and how you know.
- How they could make money with the bars that are left over after the Marathon.

4 Modules 4–6

State Capital Field Trip Plan

75 students in Grade 4 are going on an annual field trip to the state capital. 15 adults will lead a number of student groups. Students pay $4 each and adults pay $12 each for the field trip.

School buses will drop everybody off at the Capitol Building in the morning, and then depart from the same location later that afternoon.

All student groups must visit the Capitol Building, eat lunch in the City Park (at some point), and visit one other place near the Capitol Building. This means that each group will miss out on visiting one location. Groups can walk for part of the day and take a van for part of the day.

Here is the **Mileage Chart** that shows the distances between each location:

Mileage Chart					
		End Point			
		To Capitol Building	To State Museum	To City Park (for lunch)	To Outdoor Farmer's Market
Starting Point	From Capitol Building		$\frac{1}{4}$ mile	$\frac{3}{4}$ mile	$1\frac{1}{2}$ mile
	From State Museum	$\frac{1}{4}$ mile		$\frac{1}{2}$ mile	1 mile
	From City Park (after lunch)	$\frac{3}{4}$ mile	$\frac{1}{2}$ mile		$\frac{1}{2}$ mile
	From Outdoor Farmer's Market	$1\frac{1}{2}$ miles	1 mile	$\frac{1}{2}$ mile	

Compare the amount that the adults and students pay to go on the field trip. Describe the relationship between the two amounts.

The week before the trip teachers had collected $284 for the students and $144 for the adults. How many students and how many adults still need to pay for their trip..

All groups will start their day at the Capitol Building and end their day a the Capitol Building. Any group that travels more than 2 miles total during the day can take a van for part of the day. The groups in Mr. Owada's class made these plans.

Group Number	Mr. Owada's Class Walking Plan	Total Miles Walked
1	Capitol > State Museum > Lunch > Capitol	
2	Capitol > Lunch > Farmer's Market > Capitol	

Which group can take the van?

- Fill in the Total Miles Walked on the table.
- Write which group can take the van.

4 Modules 4–6

Use the information from the **State Capital Field Trip Plan** and **Question 3** to solve.

4. For each $\frac{1}{4}$ mile walked, the class earns 10 Mileage Club Points at school. After 500 points are earned, the class receives a bonus recess. Mr. Owada's class already has 130 points. They would like to earn a bonus recess after the field trip.

Mr. Owada's class has been split into five groups.

- Each group will keep track of the number of $\frac{1}{4}$ miles they walk on their field trip.
- All five groups decide not to use a van.

Julia says they will earn 500 Mileage Club Points after the field trip. Natalie disagrees. Do you agree with Julia or Natalie? Show your thinking and explain why.

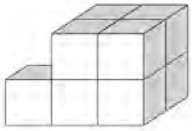
For this item you need to:

- Fill in **Total Miles Walked** and **Total Mileage Club Points** for each group on the table below.
- Decide if you agree with Julia or Natalie.
- Show why.

Group Number	Mr. Owada's Class Walking Plan	Total Miles Walked	Total Mileage Club Points
1	Capitol > State Museum > Lunch > Capitol		
2	Capitol > Lunch > Farmer's Market > Capitol		
3	Capitol > Lunch > State Museum > Capitol		
4	Capitol > Farmer's Market > Lunch > Capitol		
5	Capitol > State Museum > Lunch > Capitol		

1

How many  in this building?



HEAD POLISHERS

Yellow Tank

Thinking Mathematically and Problem Solving

2

Jose is 8 years old.
Previn is 2 years older than Jose.
Sam is younger than Jose.
When you add their ages, you get 25.

How old is Sam?

BRAIN BOOSTERS

Orange Tank

Thinking Mathematically and Problem Solving

4

This is a mixed-up multiplication table.
Copy the table.

×	3	2		
	15			20
2			10	
		6		
	12			

The top row should show the numbers 2, 3, 4 and 5.
The first column should show the numbers 2, 3, 4 and 5.
Complete the table.

MEGA MINDS

Purple Tank

Thinking Mathematically and Problem Solving

3

Binh is thinking of 2 different prime numbers.

Their sum is 30.

Their difference is 4.

What are the 2 numbers?

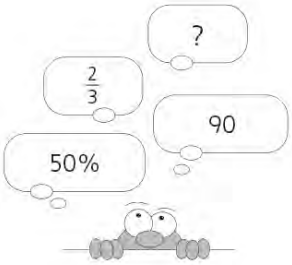
Thinking Mathematically and Problem Solving

Green Tank

2

a. How much more than **50%** of **90** is $\frac{2}{3}$ of **90**?

b. Write how you figured it out.



Thinking Mathematically and Problem Solving

Red Tank

References Cited

Dixon, J. K., Adams, T. L., Nolan, E. C., & In Kanold, T. D. (2015). *Beyond the common core: A handbook for mathematics in a PLC at work*.

Hattie, J., et. al. (2017). *Visible learning for mathematics, grades K-12: What works best to optimize student learning. Thousand Oaks, CA:* Corwin.

Hiebert, J., & Grouws, D. A. (2007). The Effects of Classroom Mathematics Teaching on Students' Learning. In F. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning*. Charlotte, NC: Information Age.

Hoffer, Wendy Ward. (2012). *Minds on mathematics: Using math workshop to develop deep understanding in Grades 4-8. Portsmouth, NH: Heineman.*

Hufferd-Ackles, K., Fuson, K., & Sherin, M. G. (2004). *Describing levels and components of a math-talk community. Journal for Research in Mathematics Education*.

Liljedahl, P. (2021). *Building thinking classrooms in mathematics, Grades K-12, 14 teaching practices for enhancing learning*. Thousand Oaks, CA: Corwin.

(2014). *Principles to actions : Ensuring mathematical success for all.* Reston, VA: NCTM: National Council of Teachers of Mathematics.

Russell, Schifter, & Bastable. (2011) *Connecting arithmetic to algebra.* Thousand Oaks, CA: Heinemann.

Russell, S.J., et.al. (2017). *But why does it work?: Mathematical argument in the elementary classroom*. Portsmouth, NH: Heinemann

Wong, H. K., Wong, R. T., & Seroyer, C. (2009). *The first days of school: How to be an effective teacher*. Mountain View, CA: Harry K. Wong Publications.