



Building Procedural Fluency from Conceptual Understanding: Multiplication and Division

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How could we figure out the cost of buying 4 hats?

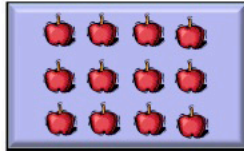
This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

What are the problem types of multiplication and division?

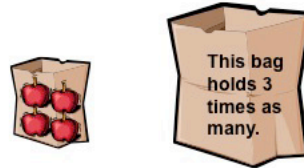
Equal Groups (Set)



Arrays (Area)



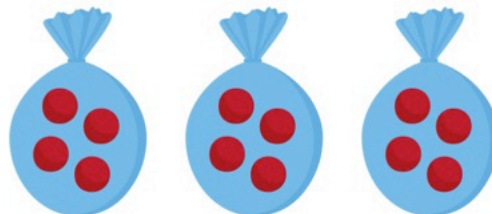
Comparison



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Equal Groups (Set) Model

Is **3 bags of 4** the same as **4 bags of 3**?



The total number of objects in the bags is the same, but the pictures are not the same.

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WHAT'S THE PROBLEM?

Carefully read each story problem.

- Check (✓) the box to indicate the operation described.
- For each multiplication problem, write whether the type is equal groups, an array or comparison.
- For each division problem, write whether the story suggests sharing or grouping.
- It is not necessary to answer the problems.

1. a. A running track is 400 meters long. How far will you run if you complete 6 laps of the track?

×	÷	?
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- b. If a team of 4 people completed one lap in a relay race, how far would each person run?

×	÷	?
---	---	---

2. a. A robot packs cans into boxes. There are 3 rows in each box and 4 cans in each row. How many cans in a box?

×	÷	?
---	---	---

- b. How many cans in 5 boxes?

×	÷	?
---	---	---

3. a. Bryce caught 4 fish. Beth caught 3 times as many. How many fish did Beth catch?

×	÷	?
---	---	---

- b. How many more fish did Beth catch than Bryce?

×	÷	?
---	---	---

4. a. It takes Ella 6 minutes to clean the snow off one car at the dealership. How many cars can she clean in an hour?

×	÷	?
---	---	---

- b. Ella has 6 cars to wash in 3 hours. How much time can she spend washing each car?

×	÷	?
---	---	---

5. a. Joel collected 40mL of water from a leaking faucet in 1 hour. How many milliliters would be collected after 3 hours?

×	÷	?
---	---	---

- b. How long would it take to collect 1 liter of water?

×	÷	?
---	---	---

Number Fact Strategies

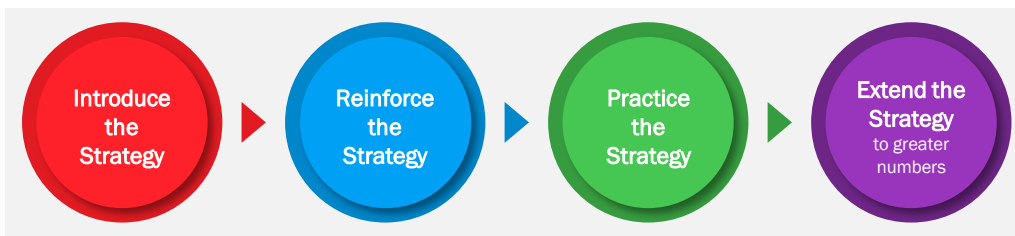
MULTIPLICATION

- Use Tens (5s facts)
- Use Doubles (2s, 4s, and 8s facts)
- Use a Rule (1s and 0s facts)
- Build Up and Build Down (9s and 6s facts)

DIVISION

- Think Multiplication

The Teaching Sequence



Tens Or Fives

$2 \times 10 = \underline{\hspace{1cm}}$ $2 \times 5 = \underline{\hspace{1cm}}$	$4 \times 10 = \underline{\hspace{1cm}}$ $4 \times 5 = \underline{\hspace{1cm}}$	$7 \times 10 = \underline{\hspace{1cm}}$ $7 \times 5 = \underline{\hspace{1cm}}$	$3 \times 10 = \underline{\hspace{1cm}}$ $3 \times 5 = \underline{\hspace{1cm}}$	$2 \times 10 = \underline{\hspace{1cm}}$ $2 \times 5 = \underline{\hspace{1cm}}$
$6 \times 10 = \underline{\hspace{1cm}}$ $6 \times 5 = \underline{\hspace{1cm}}$	$2 \times 10 = \underline{\hspace{1cm}}$ $2 \times 5 = \underline{\hspace{1cm}}$	$8 \times 10 = \underline{\hspace{1cm}}$ $8 \times 5 = \underline{\hspace{1cm}}$	$5 \times 10 = \underline{\hspace{1cm}}$ $5 \times 5 = \underline{\hspace{1cm}}$	$1 \times 10 = \underline{\hspace{1cm}}$ $1 \times 5 = \underline{\hspace{1cm}}$
$9 \times 10 = \underline{\hspace{1cm}}$ $9 \times 5 = \underline{\hspace{1cm}}$	$8 \times 10 = \underline{\hspace{1cm}}$ $8 \times 5 = \underline{\hspace{1cm}}$	$3 \times 10 = \underline{\hspace{1cm}}$ $3 \times 5 = \underline{\hspace{1cm}}$	$7 \times 10 = \underline{\hspace{1cm}}$ $7 \times 5 = \underline{\hspace{1cm}}$	$5 \times 10 = \underline{\hspace{1cm}}$ $5 \times 5 = \underline{\hspace{1cm}}$
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Cube A: 6, 5, 4, 3, 2, 1

Cube B: 9, 9, 8, 8, 7, 7

Times Tussle

20	50	25	50	10	30
70	30	10	90	45	80
35	40	25	40	15	45
80	15	50	100	90	35
45	25	20	40	50	100
45	25	30	20	30	15
70	60	35	60	20	40

REINFORCE: Double and Halve

Nice and Easy

30×3	50×3	70×3	90×3
30×4	50×4	70×4	90×4
30×6	50×6	70×6	90×6
30×7	50×7	70×7	90×7
30×8	50×8	70×8	90×8
30×9	50×9	70×9	90×9

Cube A: 15, 15, 25, 35, 45, 45

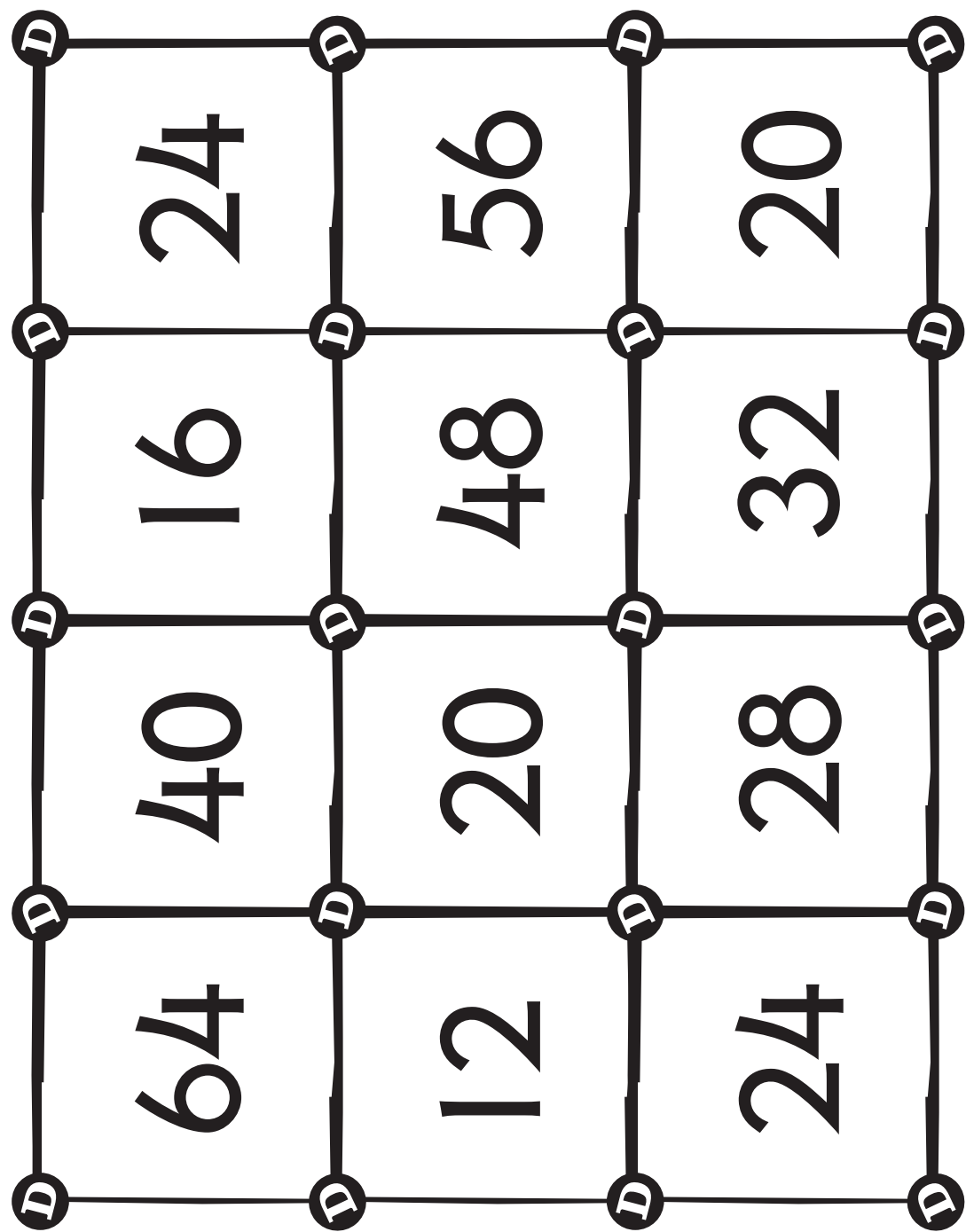
Cube B: 6, 8, 12, 14, 16, 18

Nice and Easy Too!

90	150	210	270
120	200	280	360
180	300	420	540
210	350	490	630
240	400	560	720
270	450	630	810

REINFORCE: Use Doubles

Do the *D*s



Cube A: 3, 4, 5, 6, 7, 8

Cube B: DD, DD, DD, DDD, DDD, DDD

Burnett, J. & Tickle, B. (2007). *Fundamentals: Purple Level*, page 54. ORIGO Education.

CONNECT MULTIPLICATION AND Division

Player 1

Player 2

$$24 \div \underline{\quad} = \underline{\quad}$$

$$24 \div \underline{\quad} = \underline{\quad}$$

$$20 \div \underline{\quad} = \underline{\quad}$$

$$20 \div \underline{\quad} = \underline{\quad}$$

$$18 \div \underline{\quad} = \underline{\quad}$$

$$18 \div \underline{\quad} = \underline{\quad}$$

$$16 \div \underline{\quad} = \underline{\quad}$$

$$16 \div \underline{\quad} = \underline{\quad}$$

$$15 \div \underline{\quad} = \underline{\quad}$$

$$15 \div \underline{\quad} = \underline{\quad}$$

$$12 \div \underline{\quad} = \underline{\quad}$$

$$12 \div \underline{\quad} = \underline{\quad}$$

Tally

Tally

Cube A: 2, 3, 4, 2, 3, 4

Cube B: 5, 6, 8, 5, 6, 8

Directions for the Games

Tens or Fives

Focus:

Using tens facts to multiply by five

Materials:

2 cubes with the following configuration

Cube A: 1, 2, 3, 4, 5, 6

Cube B: 7, 7, 8, 8, 9, 9

Game board

Each player will need a colored pencil or marker of a different color

Directions:

The winner is the first to build a winning sequence of four adjacent counters in a horizontal, vertical, or diagonal line, or to make a box of four.

How to Play:

Roll both cubes.

Choose to multiply the number on cube A or B by 10. Find that fact on the board and fill in the product for multiplying the chosen number by ten and by five. Four in any direction wins.

Some numbers appear more than once on the gameboard. Players must decide whether to build a winning sequence or block the other player.

If both possible numbers are not available, the player misses a turn.

Play continues until one player builds a winning sequence.

Times Tussle

Focus:

Multiplying numbers two to ten by five and ten

Materials:

3 cubes with the following configuration: cube A should be one color; B & C should be the same color as each other, but a different color than cube A.

Cube A: 5, 5, 5, 10, 10, 10

Cube B: 2, 3, 4, 5, 6, 7

Cube C: 5, 6, 7, 8, 9, 10

Each player will need 14 transparent counters (different color for each player)

Directions:

The winner is the first player to build a winning sequence four adjacent counters in a horizontal, vertical, or diagonal line.

How to Play:

Roll all cubes and choose Cube A and one other cube.

Multiply and cover the product. Four in row, any direction wins.

Some numbers appear more than once on the game board. You must decide whether to build a winning sequence or block the other player.

If both possible answers are not available, you miss a turn.

Play continues in turns until one player builds a winning sequence.

For ideas on how to bring out the mathematics in this game, see Fundamentals (Purple) pp. 28-31.

Do the Ds

Focus:

Use a doubling strategy to practice fours and eights facts.

Materials:

Do the Ds game board

One doubling cube labeled with DD on 3 faces (for double, double) and labeled with DDD on 3 faces (for double, double, double)

One cube labeled with numerals 3, 4, 5, 6, 7, 8

Four color counters for each player (a different color for each player)

Directions:

The winner is the player who is the first to place all four counters on the game board.

How to Play:

Player 1 rolls the cubes and follows the instruction, doubling the number two or three times.

The player claims the answer on the game board by covering it with a counter. If an answer is unavailable, the player misses a turn.

Each of the other players has a turn.

The first player to place all four counters on the game board is the winner.

Example:

Lily rolls 4 and DDD. She says, "Double 4 is 8, double 8 is 16, double 16 is 32. Four multiplied by 8 is 32."

Lily places her counter on the 32 and claims that space.

For ideas on how to bring out the mathematics in this game, see Fundamentals Purple (pp.52-53)

Nice and Easy

Focus:

Use the doubling-and-halving strategy to multiply

Materials:

Each pair will need: Nice and Easy Game board, one cube labeled: 15, 15, 25, 35, 45, 45 and another

Cube labeled: 6, 8, 12, 14, 16, 18

Each player will need: ten counters (a different color for each player)

Directions:

The winner is the first player to arrange three counters adjacently in a horizontal, vertical, or diagonal line.

How to Play:

Player 1 roll the cubes. Say aloud the multiplication sentence represented by the number cubes, then double one factor and half the other to figure out the equivalent multiplication sentence. Place a counter on the matching multiplication sentence on the game board. Calculate and say the product. If your sentence is not available then you loss a turn.

The other player has a turn.

The first player to make a line of three adjacent counters is the winner.

Example:

Lincoln rolls 35 and 16, and says: 35 multiplied by 16 is the same as 70 multiplied by 8. The answer is 560. He claims 70 x 80 on the game board.

For ideas on how to bring out the mathematics in this game, see Fundamentals Red (pp.20-23)

Connect Multiplication and Division

Focus:

Use multiplication to divide

Materials:

Two number cubes configured as follows:

Cube A: 2, 3, 4, 2, 3, 4

Cube B: 5, 6, 8, 5, 6, 8

Take or Tally Game board

Directions:

The first player to complete his/her side before receiving 5 tallies is the winner.

How to Play:

First player rolls both cubes.

Player writes the two numbers in one of the number sentences on his/her game board. The completed equation must be true.

If a true sentence cannot be made, the player makes a tally in the space provided at the bottom of the game board.

The first player to complete 6 equations before making 5 tallies is the winner.

Multiplication and Division Strategies Videos

Introducing the ORIGO Model for Teaching Skills

ORIGO One: <https://origo-education.wistia.com/medias/26icnyoznj>

Teaching the Use-Tens Strategy for Multiplication

ORIGO One: <https://origo-education.wistia.com/medias/gyw3z0dyld>

GS9: Exploring a strategy to Multiple by Five

Gem Stones: <https://www.youtube.com/watch?v=g0qNTyY8yz8>

GS8: Using arrays to explore turn around facts for multiplication

Gem Stones: <https://youtu.be/WhXryzUmX5Y>

Teaching the Doubling Strategy for Multiplication

ORIGO One: <https://origo-education.wistia.com/medias/ikn1y886en>

Teaching the Build-Up Strategy for Multiplication

ORIGO One: <https://origo-education.wistia.com/medias/17saqfy40q>

Teaching the Build-Down Strategy for Multiplication

ORIGO One: <https://origo-education.wistia.com/medias/gtwhame631>

GS21: Building Down from a Known “tens” Facts to Multiply by 9

Gem Stones: <https://www.youtube.com/watch?v=GDwhFy5PsGM>

Teaching the Think-Multiplication Division Strategy

ORIGO One: <https://origo-education.wistia.com/medias/aOn12cd5d2>

