

### A Clear Vision for Utilizing Number Lines

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### Round 540 to the Nearest Hundred





### Aims of Session

Participants will **understand** that...

- there is a <u>progression</u> to utilizing number lines in classrooms
- using targeted <u>language</u> and <u>discourse</u> helps students make sense of number lines

Participants will know...

• the <u>benefits</u> of using number lines as models

Participants will be able to ...

 plan <u>engaging activities</u> help students connect mathematical concepts to the number line model



### Why teach with models?

Representations are "useful tools for building and communicating both information and understanding."

NCTM, 2000

The foundations of mathematics rest on the core representations that we have **internalized**.

Dehaene, 1997

Linking of representations "enables us to see complex ideas in a new way and apply them more effectively."

Kaput, 1998



### Models are used to...

- help <u>students</u> develop new concepts or relationships.
- help <u>students</u> make connections between concepts and symbols.
- help <u>teachers</u> assess students' understanding.



### Why number lines as models?

Number lines may be "the most important construct within the realm of number and operation."

Frykholm, 2010

What are the benefits of using number lines as mathematical models?



# Why number lines as models?

- Represent multiple types of numbers
- Connect to counting, operations, and relationships
- Promote creative solution strategies and intuitive reasoning
- Increase cognitive engagement and decrease memory load
- Lead to better understanding and performance in later years
- Make thinking visible



### Where do we begin?

Students need experiences with number tracks to fully understand the abstract idea of a number line.

Number tracks serve to bridge discrete set models and the continuous number line model.



### Where do we begin?

Build a number track.



# Note that zero (0) would not have a space on a number track.



# Build a Number Track



- Put a finger on 5, put another finger on 7. What number comes between 5 and 7?
- Break apart all the cubes and put the numbers in order.
- What number comes just after five?





# Build a Number Track



- Take a cube away and ask a friend to name the missing number.
- What number do you land on if you start at 5 and jump on 2 more?
- Turn over every second cube. Read the numbers.





### Use Language and Discourse



















### Use Language and Discourse





### Use Number Tracks







### **Build a Hundred Chart**

I	2	3	4	5	6	7	8	9	Ι	2	3	4	5	6	7	8	9	10	22	23	24	25	26	27	28	29	30
									Ш	12	13	14	15	16	17	18	19	20									
									21	22	23	24	25	26	27	28	29	30									
									21	22	23	24	25	26	27	28	29	30									
									41	42	43	44	45	46	47	48	49	50									
									51	52	53	54	55	56	57	58	59	60									
									61	62	63	64	65	66	67	68	69	70									
									71	72	73	74	75	76	77	78	79	80									
									81	82	83	84	85	86	87	88	89	90									
									91	92	93	94	95	96	97	98	99	100									



### What is wrong with this picture?

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99





Start 	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	<mark>43</mark>	<mark>44</mark>	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

First to 100

+	+12	+12
+13	+21	+21
+22	+23	+23
+31	+32	+32
+33	-10	-10
-10	-20	-20
-20	-30	-30
	+11 +13 +22 +31 +33 -10 -20	+11 +12 +13 +21 +22 +23 +31 +32 +33 -10 -10 -20 -20 -30





### First to 100

Adding on a hundred chart

### Purpose

In this game, students practice adding two-digit numbers by breaking up one addend into tens and ones. A hundred chart is used to help the students add the tens and then the ones. For example, to calculate 43 + 24, the students are encouraged to think, 43 + 20 + 4.

### Materials

Each pair of players will need

- A 'First to 100' game board (page 18) as shown below.
- One (1) set of operation cards. Copy page 19 as shown below. Cut out and laminate the cards to make one set.

### Each player will need

 One (1) transparent counter (a different color for each player),



### How to Play

The aim is to reach or pass 100 on the game board.

 The operation cards are shuffled and placed face down in a stack.

2 players

- The players place their counters in the 'Start' position

   on the game board.
- The first player draws the top card. If an addition card is drawn, the player moves his or her counter that number of spaces forward on the game board.

Example: Katie draws '+ 12' and moves her counter down one row (+ 10) and two spaces to the right (+ 2).

 Initially, if a subtraction card is drawn, the player remains at 'Start'. Throughout the game, if a subtraction card is drawn, the player moves his or her counter back that number of spaces.

### Example: Jason draws '- 20' and moves his counter up two rows (- 20).

- If a subtraction card would result in a score less than 1, the player returns his or her counter to 'Start'.
- The card is placed at the bottom of the stack.
- The other player has a turn.
- The first player to reach or pass 100 is the winner.

### **Reading the Research**

Hundred boards can be a useful model for helping children see how higher-decade addition and subtraction facts are related. They also help children see patterns and relationships. For example, 8 + 7 begins at 8 and ends on 15 on the next row. Similarly, the sums 38 + 7, 48 + 7, ... 88 + 7 all begin and end in the same columns (Van de Walle & Bowman Watkins, 1993).

Fundamentals

### Before the Game

Show a transparency of the game board on the overhead projector. Place a transparent counter on 54. Select an addition card such as '+ 32' and ask students to figure out the sum. Encourage them to explain how they calculated the answer. For example, can the students see that they need to move the counter down three rows and two spaces to the right? Do they know that this is the same as adding 30 and then adding 2? Discuss alternative strategies, such as adding the ones first, followed by the tens.

### During the Game

At various stages of the game, ask students how many more they need to make 100. Encourage them to share how they figured out the answer. The discussion will vary, but they should see that one efficient strategy is to build the ones to the next multiple of ten and then figure out the number of tens they need to get to 100. For example, if a player has a counter on 62, he or she will need 8 more to make 70 and 30 more to make 100, so the player needs a total of 38. Alternatively, they could count forward in tens and then build the ones to 100.

### After the Game

Ask students to explain a general method of finding out how many more they need to make 100. A student might say, I figure out how many more ones I need to get to the next multiple of ten and then see how many more tens I need to get to 100.

Use a transparency of the game board on the overhead projector to pose hypothetical questions. For example, place a counter on the number 83 and ask what the total would be if '+ 23' was drawn from the stack of cards. Can the students see that the same strategies work for moves beyond 100?

Have the students calculate similar sums without the aid of the game board. Ask them to explain their methods. For example, write 51 + 42 on the board and ask how they could calculate the answer. A student might say, 51 + 40 = 91 and 91 + 2 = 93. Can the students see that this is a much more efficient strategy than counting by ones?

### **Beyond the Game**

Give the students square grid paper to make a game board for numbers beyond 100, such as 301-400. Using the operation cards from 'First to 100', the students can race to a new target, for example 400. The game rules remain the same.





Start 401	402	403	404	405	406	407	408	409	410	
411										
421										
431										
441										
451										
461										
<mark>47</mark> 1										
481										
491									500	

Break It Down

+	+12	+12
+13	+21	+21
+22	+23	+23
+31	+32	+32
+33	-10	-10
-10	-20	-20
-30	-30	-30
	+11 +13 +22 +31 +33 -10 -30	+11 +12 +13 +21 +22 +23 +31 +32 +33 -10 -10 -20 -30 -30



6	Cards										
	Curus	1.10	1.09	1.08	1.07	1.06	1.05	1.04	1.03	1.02	1.01
	+0.32	1.20	1.19	1.18	1.17	1.16	1.15	1.14	1.13	1.12	1.11
		1.30	1.29	1.28	1.27	1.26	1.25	1.24	1.23	1.22	1.21
		1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.33	1.32	1.31
	Counters - Player 1	1.50	1.49	1.48	1.47	1.46	1.45	1.44	1.43	1.42	1.41
		1.60	1.59	1.58	1.57	1.56	1.55	1.54	1.53	1.52	1.51
		1.70	1.69	1.68	1.67	1.66	1.65	1.64	1.63	1.62	1.61
	Counters - Player 2	1.80	1.79	1.78	1.77	1.76	1.75	1.74	1.73	1.72	1.71
	Counters Proj	1.90	1.89	1.88	1.87	1.86	1.85	1.84	1.83	1.82	1.81
2		2	1.99	1.98	1.97	1.96	1.95	1.94	1.93	1.92	1.91
	Fundamentals										



0000

First to Two

					-	-			
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.21	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.30
0.31	0.32	0.33	0.34	0.35	0.36	0.37	0.38	0.39	0.40
0.41	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50
0.51	0.52	0.53	0.54	0.55	0.56	0.57	0.58	0.59	0.60
0.61	0.62	0.63	0.64	0.65	0.66	0.67	0.68	0.69	0.70
0.71	0.72	0.73	0.74	0.75	0.76	0.77	0.78	0.79	0.80
0.81	0.82	0.83	0.84	0.85	0.86	0.87	0.88	0.89	0.90
0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	Т





### Step In Number: Introducing number lines and representing numbers as lengths from zero

Look at the number track.

What number would you write in the position that is shaded? How do you know?



Look at the number line above.

How is it the same as the number track? How is it different? Where should we write **0** on the number line?

Which mark on the number line shows the same number that is shaded on the number track? How do you know?

What is a quick way to find 17 on the number line?



### Use Language and Discourse







ΕD

ATION









Cube A: 25, 27, 29, 35, 36, 38

**Cube B:** 10, 11, 12, 13, 14, 15

The aim is to achieve the greater difference.

- The first player rolls the number cubes.
- Using the two numbers, the player writes a subtraction sentence in the first round on his or her game board.

### Example: Beth rolls 13 and 36, and writes 36 – 13 = \_\_\_\_.

 The player mentally calculates the answer and writes it in the number sentence, then uses the number line to show his or her thinking.

Example: Beth writes 36 – 13 = 23 and shows how she starts with 36, counts back 10, then 3.





Leaps and Bounds

- The other player(s) has a turn.
- The player with the greater number of points after seven rounds is the winner.







Jump Back

**Cube A:** 166, 167, 177, 178, 188, 189

**Cube B:** 23, 24, 25, 33, 34, 35



The aim is to achieve the greater difference.

- The first player rolls the number cubes.
- Using the two numbers, the player writes a subtraction sentence in the first round on his or her game board.

Example: Silas rolls 34 and 178. He writes 178 - 34 =\_\_\_\_.

 The player mentally calculates the answer and writes it in the number sentence, then uses the number line to show his or her thinking.

Example: Silas writes 178 – 34 = 144 and shows how he starts with 178, counts back 30, then 4.



- The other player(s) has a turn.
- The player with the greater number of points after five rounds is the winner.









### Comparison





Take Away





Comparison





### Raise the Level of Open Number Lines





# Number Line Match



Draw a line from each expression to the number line that can be used to model the operation.

**Hint:** Some expressions can be matched to more than one number line.





### Number Line Match





### **Build on Counting Strategies**





### **Build on Counting Strategies**



38 + 25









Students need to:

- move from using multiple copies of a unit of length to work out "how many fit" to
- the idea that you can use one copy of the rod and "mark it off" along the thing to be measured to
- making a tape measure using a unit.







### Move to the Number Line



















400 - 265





400 - 265





400 - 265 435 - 300





**Cube A:** 92, 82, 72, 62, 52, 42

**Cube B:** 17, 27, 37, 17, 27, 37



### **Comparison Subtraction**



"Can I do it my way?"

600 – 1	599
432 – 1	- 431
	168



### Reflection

What actions will you take in the upcoming year as a result of today's learning?



### Aims of Session

Participants will **understand** that...

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- using targeted <u>language</u> and <u>discourse</u> helps students make sense of number lines

Participants will know...

• the <u>benefits</u> of using number lines as models

Participants will be able to ...

 plan <u>engaging activities</u> help students connect mathematical concepts to the number line model





### Thank You



